Sanskrit /n/-retroflexion is one of the most complex segmental processes in phonology. While it is still star-free, it does not fit in any of the subregular classes that are commonly entertained in the literature. We show that when construed as a phonotactic dependency, the process fits into a class we call input-output tier-based strictly local (IO-TSL), a natural extension of the familiar class TSL. IO-TSL increases the power of TSL’s tier projection function by making it an input-output strictly local transduction. Assuming that /n/-retroflexion represents the upper bound on the complexity of segmental phonology, this shows that all of segmental phonology can be captured by combining the intuitive notion of tiers with the independently motivated machinery of strictly local mappings.

1 Introduction

Subregular phonology seeks to identify proper subclasses of the finite-state languages and transductions that are sufficiently powerful for natural language phenomena (see Heinz 2018 and references therein). In addition to establishing tighter bounds on cross-linguistic variation, many of these subclasses are also efficiently learnable in the limit from positive text (Heinz et al., 2012; Jardine and McMullin, 2017).

Sanskrit /n/-retroflexion, also called nati, is noteworthy because it has been known for a long time to be subregular but to occupy a very high position in the subregular hierarchy when construed as a phonotactic dependency (Graf, 2010; Jardine, 2016). Its singularly high complexity stems from the combination of a locally specified target (/n/ immediately before a sonorant) with both a non-local trigger (a preceding retroflex) and three independent blocking effects, one of which is itself subject to blocking. Established classes such as strictly local (SL) and its extension tier-based strictly local (TSL; Heinz et al., 2011) cover a wide range of phonological phenomena, yet they provably cannot enforce the phonotactic conditions of nati.

However, as we show in this paper, nati can be handled by a natural extension of TSL. In TSL, a tier projection function masks out all segments that do not belong to some specified subset of the alphabet. This allows for simple non-local dependencies to be regulated in a local fashion. More involved patterns can be accommodated by increasing the complexity of the tier projection. In order to capture nati, the projection function has to consider two factors when choosing whether or not to project a symbol: i) the local context in the string, and ii) which symbols are already on the tier. This makes it a special case of input-output strictly local maps, which is why we call this extended version of TSL input-output TSL (IO-TSL).

IO-TSL is a natural extension of TSL — it subsumes it as a special case and expands on recent proposals to make tier projection structuresensitive. De Santo and Graf (2017) propose input strictly local maps to handle certain cases noted as problematic for TSL in McMullin (2016), and similar proposals are made in Baek (2017) and Yang (2018) for phonology and Vu et al. (2018) for syntax. Mayer and Major (2018), on the other hand, suggest based on Graf (p.c.) that backness harmony in Uyghur is TSL with output strictly local tier projection; Graf and Shafiei (2018) apply the same idea to syntax. Input-output strictly local projection merely combines these two extensions.

The paper is laid out as follows. We first introduce TSL (§2.1) and subsequently generalize it to IO-TSL (§2.2), some properties of which are discussed in §2.3. The empirical facts of nati are presented in §3 based primarily on Ryan (2017), followed by our IO-TSL analysis in §4.
2 Defining IO-TSL

2.1 TSL

Throughout the paper, we use $\varepsilon$ to denote the empty string, $S^*$ for the Kleene closure of $S$, and $S^+$ for $S^*$ without the empty string. We use $S^k$ to denote the proper subset of $S^*$ that only contains strings of length $k$, and we write $s^k$ as a shorthand for $\{s\}^k$.

Let $\Sigma$ be some fixed alphabet and $s \in \Sigma^*$. The set $f_k(s)$ of $k$-factors of $s$ consists of all the length-$k$ substrings of $s^{k-1} \times s^{k-1}$, where $\varepsilon, \kappa \not\in \Sigma$ and $k \geq 1$.

**Definition 1.** A stringset $L \subseteq \Sigma^*$ is strictly $k$-local (SL-$k$) iff there is some $G \subseteq (\Sigma \cup \{\varepsilon, \kappa\})^k$ such that $L = \{s \in \Sigma^* \mid f_k(s) \cap G = \emptyset\}$.

Intuitively, $G$ defines a grammar of forbidden substrings that no well-formed string may contain. The class SL of strictly local stringsets is $\bigcup_{k \geq 1}$ SL-$k$.

**Example 1.** The string language $(ab)^+$ is generated by the grammar $G := \{\varepsilon\} \cup \{b, aa, bb, \varepsilon a\}$ and is thus SL-2. For instance, $aba$ is illicit because $f_2(aba) \cap G = \{a\varepsilon\} \neq \emptyset$, whereas $f_2(abab) \cap G = \emptyset$.

For every $T \subseteq \Sigma - \{\varepsilon\}$, a simple tier projection $\pi_T$ is a transduction that deletes all symbols not in $T$:

$$\pi_T(\sigma u) := \begin{cases} \varepsilon & \text{if } \sigma u = \varepsilon \\ \sigma \pi_T(u) & \text{if } \sigma \in T \\ \pi_T(u) & \text{otherwise} \end{cases}$$

**Definition 2.** A stringset $L \subseteq \Sigma^*$ is tier-based strictly $k$-local (TSL-$k$) iff there exists a $T \subseteq \Sigma - \{\varepsilon\}$ and an SL-$k$ language $K \subseteq T^*$ such that $L := \{s \in \Sigma^* \mid \pi_T(s) \in K\}$. It is TSL iff it is TSL-$k$ for some $k$.

TSL languages are string languages that are SL once one masks out all irrelevant symbols.

**Example 2.** Consider all strings over $\{a, b, c\}$ that contain exactly one $b$ and exactly one $c$. This language is TSL-3: let $T := \{b, c\}$, and $K := \{bc, cb\}$, which is an SL-3 language (the reader is invited to write down the grammar for $K$). The licit string $aabac$, for instance, is first projected to $bc$, which is a member of $K$. The illicit $aaba$, on the other hand, is projected to $b \not\in K$.

2.2 IO-TSL

The power of TSL can be increased by changing the nature of the tier projection $\pi$. In particular, it can be generalized to strictly local maps (Chandlee, 2014). Due to space constraints, we immediately define input-output strictly local projections without discussing the earlier work on subregular mappings on which our idea builds. The interested reader should consult Chandlee (2014, 2017) and Chandlee and Heinz (2018).

An $(i, j)$-context $c$ is a 4-tuple $\langle \sigma, b, a, t \rangle$ with $\sigma \in \Sigma$, $t$ a string over $\Sigma \cup \{\varepsilon\}$ of length $j - 1$, and $a$ and $b$ strings over $\Sigma \cup \{\varepsilon, \kappa\}$ of combined length $i - 1$. The context specifies that $\sigma$ should be projected whenever both of the following hold: it occurs between the substrings $b$ (look-back) and $a$ (look-ahead), and the tier constructed so far ends in $t$. The value of $i$ determines how far back along the tier we can look, including the current symbol. The value of $j$ indicates how far back along the tier we can look, including the current symbol. Given a set of contexts $c_1, c_2, \ldots, c_n$, we call it an $(i, j)$-context set $C(i, j)$ iff for every $c_m (1 \leq m \leq n)$ there are $i_m \leq i$ and $j_m \leq j$ such that $c_m$ is an $(i_m, j_m)$-context.

Note that in a context set $C(i, j)$, $i$ and $j$ refer to the maximum input and output window sizes considered by any $(i, j)$-context. The individual $(i, j)$-contexts may vary in size within these bounds. This is merely a matter of notational convenience and does not affect generative capacity.

**Definition 3.** Let $C$ be an $(i, j)$-context set. Then the input-output strictly $(i, j)$-local (IOSL-$i, j$) tier projection $\pi_C$ maps every $s \in \Sigma^*$ to $\pi_C(s^{i+1} \times \Sigma^*)$, where $\pi_C((ab \ast \sigma av, wt))$ is

$$\begin{cases} \varepsilon & \text{if } \sigma av = \varepsilon, \\ \sigma \pi_C((ub \ast \sigma av, wt)) & \text{if } \langle \sigma, b, a, t \rangle \in C, \\ \pi_C((ub \ast \sigma av, wt)) & \text{otherwise}. \end{cases}$$

for $\sigma \in \Sigma$ and $a, b, t, u, v, w \in (\Sigma \cup \{\varepsilon, \kappa\})^*$.

The first argument to $\pi_C'$ is the input string, with $\ast$ as a diacritic to mark the position up to which the string has been processed. The second argument contains the symbols that have already been projected. A schematic diagram of the projection function $\pi_C'$ that arises from $\pi_C$ is shown in Fig. 1.

**Example 3.** Let $\Sigma := \{a, b, c\}$ and consider the tier projection that always projects the first and last symbol of the string, always projects $a$, never projects $c$, and projects $b$ only if the previous symbol on the tier is $a$. This projection is IOSL-(2,2).

The context set contains all the contexts below, and only those:
Conjecture 1. IO-TSL ⊊ Star-Free.

Consider the star-free string language $L := aL'a ∪ bL'b$ where $L'$ is $(d^+cd^+ed^+)^+$. In order to ensure the matching of the first and last segment those have to be projected too. But then the set of well-formed tiers is $a(ce)^+a ∪ b(ce)^+b$, which is not in SL because it violates suffix substitution closure (cf. Heinz, 2018). Hence $L$ is not IO-TSL (although it is in the intersection closure of TSL). A fully worked out proof would have to show that all other IOSL tier projections fail as well.

Like most subregular language classes, IO-TSL is not closed under relabeling. This follows from the familiar insight that $(aa)^+$, which isn’t even star-free, is a relabeling of the SL-2 language $(ab)^+$. We state a few additional conjectures without further elaboration.

Conjecture 2. IO-TSL is not closed under intersection, union, relative complement, or concatenation.

Conjecture 3. IO-TSL is incomparable to the following classes:

- locally threshold testable languages (LTT),
- locally testable (LT),
- piecewise testable (PT),
- interval-based strictly piecewise (IBSP; Graf, 2017, 2018)
- strictly piecewise (SP: Rogers et al., 2010)

If correct, these properties of IO-TSL would mirror exactly those of TSL, further corroborating our claim that IO-TSL is a very natural generalization of TSL.

That said, IO-TSL is a fair amount more complex than TSL. In the next section, we discuss the empirical facts of Sanskrit /n/-retroflexion that motivate the introduction of this additional complexity.

3 Sanskrit /n/-retroflexion

Sanskrit /n/-retroflexion, also called nati, has been studied extensively throughout the history of linguistics, and has received particularly close scrutiny within generative grammar. The notorious complexity of the phenomenon is the product of the interaction of multiple (individually simple) conditions: long-distance assimilation (§3.1), blocking by preceding coronals (§3.2), mandatory adjacency to sonorants (§3.3), blocking by preceding plosives (§3.4), and blocking by following retroflexes (§3.5).
Even a cursory look at the previous literature is beyond the scope of this paper, so we refer the reader to Ryan (2017) for a detailed literature review and analysis of the phenomenon. We draw data from Müller (1886), Hansson (2001), and Ryan (2017) and use the transcription conventions from Ryan (2017). Page numbers for the sources are indicated in the table captions of the data.

### 3.1 Base pattern

The central aspect of nati is simple: underlyingly anterior /n/ becomes retroflex [n] when it is preceded in the word by a non-lateral retroflex continuant (one of /tʃ/, /lʃ/, /kʃ/, or /hʃ/). The retroflex trigger can occur arbitrarily far to the left of the nasal target. Tables 1 and 2 respectively show the alternations in the instrumental singular suffix /-eːna/ when attached to roots without and with a trigger. Triggers, blockers, and targets are bolded in all tables.

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>kám-eːna</td>
<td>‘by desire’</td>
</tr>
<tr>
<td>bāṇ-eːna</td>
<td>‘by arrow’</td>
</tr>
<tr>
<td>muːqʰ-eːna</td>
<td>‘by the stupid (one)’</td>
</tr>
<tr>
<td>jog-eːna</td>
<td>‘by means’</td>
</tr>
</tbody>
</table>

Table 1: Forms with no nati (Ryan, 2017, p. 305)

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>naːq-eːna</td>
<td>‘by man’</td>
</tr>
<tr>
<td>maṇusj-eːna</td>
<td>‘by human’</td>
</tr>
<tr>
<td>qagʰau-eːna</td>
<td>‘by the Rāghava’</td>
</tr>
<tr>
<td>pusuːgaqʰ-eːna</td>
<td>‘by the heap of flowers’</td>
</tr>
<tr>
<td>bhaːfewaːnja</td>
<td>‘kind to Brahmans’</td>
</tr>
<tr>
<td>niːmaːna</td>
<td>‘rested’</td>
</tr>
<tr>
<td>aːkaɾaːvat</td>
<td>‘having eyes’</td>
</tr>
</tbody>
</table>

Table 2: Basic nati examples (Ryan, 2017, p. 305 and Müller, 1886, p. 44)

Viewing nati as a phonotactic phenomenon rather than a mapping from underlying representations to surface forms, we can formalize it as the constraint that no [n] may appear in the context $R\cdots$, where $R$ is a non-lateral retroflex continuant. This does not constitute an analysis, but it clarifies the formal character of the process. As we will see in the remainder of this section, though, the context is in fact much more complicated than just $R\cdots$.

### 3.2 Unconditional blocking by intervening coronals

If a coronal segment (including retroflexes) occurs between the trigger and the target, then nati is blocked. The only exception to this is the palatal glide /j/ (cf. Table 2) — this is an important point that we will return to in §4.3. Crucially, [n] itself is a coronal blocker, meaning the assimilation process only affects the first in a series of eligible targets. An exception to this is geminate /mn/ sequences, where both instances of /n/ undergo nati (cf. Table 2; this complication will also be discussed in §4.3). Examples of nati blocked by an intervening coronal are shown in Table 3. Leaving aside geminates for the moment, the illicit context for [n] now becomes $R\overline{C}$, where $C$ matches [j] and all segments that are not coronals.

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>qatʰ-eːna</td>
<td>‘by chariot’</td>
</tr>
<tr>
<td>gaːqː-eːna</td>
<td>‘by Garuḍa’</td>
</tr>
<tr>
<td>pʰaːqːniːja</td>
<td>‘lead forth’</td>
</tr>
<tr>
<td>vaːqːanaːnam</td>
<td>no gloss</td>
</tr>
</tbody>
</table>

Table 3: Intervening coronal blocking (Hansson, 2001, p. 227 and Ryan, 2017, p. 305)

### 3.3 Mandatory adjacency to sonorant

Next, the /n/ must be immediately followed by a non-liquid sonorant to undergo nati. More precisely, the following symbol must be a vowel, a glide, /m/, or /n/ itself (Whitney, 1889). No other nasals can occur following /n/ due to independent phonotactic constraints in the language (Emeneau, 1946). Like the special status of /j/ and geminates, this will become important in §4.3 but can be ignored for now.

Examples of cases where nati is blocked by the following sound, or lack thereof, are shown in Table 4. Again updating the illicit context for [n], we get $R\overline{C}^\ast S$, where $S$ is a vowel, glide, /m/, or /n/.

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>bhaːfima</td>
<td>‘brahman’</td>
</tr>
<tr>
<td>tʃʰ-n=ːt-ːe</td>
<td>‘split (3Pl middle)’</td>
</tr>
<tr>
<td>caːɾ-aːɾ-tʰi</td>
<td>‘wander (3Pl)’</td>
</tr>
</tbody>
</table>

Table 4: Blocking when no non-liquid sonorant follows (Hansson, 2001, p. 229 and Ryan, 2017, p. 318)
3.4 Conditional blocking by preceding velar and labial plosives

In addition to coronals blocking when they intervene between the trigger and the target, velar and labial plosives can also block nati, but only when two conditions are met at the same time: i) the plosive occurs immediately before the target, and ii) a left root boundary \( \sqrt{\cdot} \) intervenes between the target and trigger. Left root boundaries are generally omitted for clarity when they occur at the left edge of a word. Table 5 shows that left root boundaries alone are not sufficient to block nati. Table 6 shows cases where a labial or velar plosive blocks nati across a left root boundary, and Table 7 shows cases where such a plosive does not block because no left root boundary intervenes.

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p\alpha-a-t\acute{\imath} )</td>
<td>‘vanishes (3s)’</td>
</tr>
<tr>
<td>( t\acute{\imath}a-t\acute{\imath}a )</td>
<td>‘V( \acute{\imath} )tra-killing’</td>
</tr>
<tr>
<td>( p\alpha-a-t\acute{\imath}a-t\acute{\imath}i )</td>
<td>‘frustrates (3s)’</td>
</tr>
</tbody>
</table>

Table 5: Nati occurring across left root boundaries (Ryan, 2017, pp. 320, 321, 324)

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p\alpha-p\acute{\imath}o-t\acute{\imath}i )</td>
<td>‘attains (3s)’</td>
</tr>
<tr>
<td>( p\alpha-p\acute{\imath}o-nu-\acute{\imath}h )</td>
<td>‘should attain (2s opt.)’</td>
</tr>
<tr>
<td>( p\alpha-b\acute{\imath}ag-n\acute{\imath}a )</td>
<td>‘broken’</td>
</tr>
</tbody>
</table>

Table 6: Labial/velar blocking with intervening root boundary (Ryan, 2017, pp. 318, 321)

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t\acute{\imath}ug-\acute{\imath}a )</td>
<td>‘break (pass. part.)’</td>
</tr>
<tr>
<td>( t\acute{\imath}p-\acute{\imath}V )</td>
<td>‘be satisfied (pres. stem)’</td>
</tr>
<tr>
<td>( t\acute{\imath}i-k\acute{\imath}a-s )</td>
<td>‘inheritance’</td>
</tr>
</tbody>
</table>

Table 7: No labial/velar blocking (Ryan, 2017, p. 319)

To accommodate these new facts, we change the context to Ro\_S. Here \( \alpha \) is any string that does not contain a coronal and does not match \( \ldots \) \( \cdot \) \( \cdot \ldots \; P \), where \( P \) is a velar plosive or a labial plosive.

3.5 Conditional blocking by following retroflex

There is one final complication: nati is blocked when a retroflex occurs somewhere to the right of the target /\( n/\). Like with blocking by plosives, though, two conditions must be met: i) as above, a left root boundary separates the trigger and the target of nati, and ii) no coronal intervenes between /\( n/\) and the blocking retroflex (so coronals “block” retroflex blocking). Keep in mind that, as in §3.2, /\( n/\) itself is coronal and thus can act as a blocker, a point that will be important in §4.

Table 8 shows cases where a following retroflex blocks in the presence of a left root boundary intervening between trigger and target, and Table 9 shows cases where it does not block because a coronal intervenes between the target and the blocker, or no intervening boundary is present. Note in the final example of Table 9 that an intervening left root boundary between the trigger and the blocker has no effect — the boundary must occur before the targeted /\( n/\).

Ryan (2017) notes that it is unclear from the data whether the retroflex blocking is truly unbounded, or if the blocker must occur within a certain distance of the target. We assume the pattern is unbounded here, but it does not significantly alter the analysis if this is not the case.

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p\alpha-\eta\acute{\imath}t\acute{\imath}um )</td>
<td>‘to vanish (inf.)’</td>
</tr>
<tr>
<td>( p\alpha-\eta\acute{\imath}t )</td>
<td>‘dance forth’</td>
</tr>
<tr>
<td>( p\alpha-nak\acute{\imath} )</td>
<td>‘approach’</td>
</tr>
</tbody>
</table>

Table 8: Following retroflex blocking (Ryan, 2017, p. 325)

<table>
<thead>
<tr>
<th>Form</th>
<th>Gloss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p\alpha-\eta\acute{\imath}t\acute{\imath}t )</td>
<td>‘leader’</td>
</tr>
<tr>
<td>( b\acute{\imath}\acute{\imath}ma\acute{\imath}t\acute{\imath} )</td>
<td>‘Brahmins (loc. pl.)’</td>
</tr>
<tr>
<td>( p\alpha-k\acute{\imath}t\acute{\imath} )</td>
<td>‘unite (2s)’</td>
</tr>
<tr>
<td>( p\alpha-k\acute{\imath}t\acute{\imath} )</td>
<td>‘ancient rishi’</td>
</tr>
</tbody>
</table>

Table 9: No following retroflex blocking (Ryan, 2017, p. 325, 326)

With this last piece of data in place, we can finally give a full description of nati as a phonotactic constraint on the distribution of /\( n/\) in surface forms.

**Definition 5** (nati). No /\( n/\) may occur in a context Ro\_S\( \beta \) such that the following hold:

- \( R \) is a non-lateral retroflex continuant, and
- \( S \) is a vowel, glide, /m/, or /n/ and
- none of the following blocking conditions are met:
- \( \alpha \) contains a coronal \( C \), or
- \( \alpha \) matches \( \cdots \sqrt{\cdots P} \), where \( P \) is a velar plosive or labial plosive, or
- \( \alpha \) contains \( \sqrt{\cdot} \), and \( \beta \) contains a retroflex that is not preceded by a coronal in \( \beta \).

This description can be translated into a first-order formula with precedence to show that \( \text{nat}i \) is star-free. In the next section, we show that it is also IO-TSL.

4 Formal analysis

The IO-TSL analysis of \( \text{nat}i \) is a bit convoluted, but more straightforward than one might expect. All the heavy lifting is done by the tier projection mechanism (§4.1). In any case where the projection considers the context at all, it uses a look-ahead of 1 or 2, a look-back of 1 in the string, a look-back of 1 on the tier, or a mixture of these options. In particular, \( P \) and \( C \) have complex tier projection conditions. Our projection function creates tiers of a very limited shape that are easily shown to be SL-3 (§4.2). While our analysis uses abstract symbols such as \( R, P, \) and \( C \), the complexity of \( \text{nat}i \) remains the same even if one talks directly about the relevant segments instead (§4.3).

4.1 Tier projection

As contexts make for a verbose description, we opt for a more informal specification of the IOSL tier projection. The projection rules for each symbol are sufficiently simple that this does not introduce any inaccuracies.

An IOSL-(3, 2) tier projection for \( \text{nat}i \) is shown below. For each condition we list its individual complexity. Note that the rules below merely specify how tiers are constructed, not which tiers are well-formed. This is left for §4.2.

- Always project \( R \).  
  \( \text{IOSL-(1,1)} \)
- Project \( S \) if it is immediately preceded by \([n]\) in the input.  
  \( \text{IOSL-(2,1)} \)
- Project \( \sqrt{\cdot} \) if the previous tier symbol is \( R \).  
  \( \text{IOSL-(1,2)} \)
- Project \( P \) if the previous tier symbol is \( \sqrt{\cdot} \) and the next two input symbols are \([n]\) and \( S \).  
  \( \text{IOSL-(3,2)} \)
- Project \( C \) if the previous tier symbol is \( R \), \( \sqrt{\cdot} \), or \( S \), unless \( C \) is \([n]\) and the next input symbol is \( S \).  
  \( \text{IOSL-(2,2)} \)
- Project every retroflex (not just those matching \( R \)) if the previous tier symbol is \( S \).  
  \( \text{IOSL-(1,2)} \)
- Don’t project anything else.  
  \( \text{IOSL-(1,1)} \)

Table 10 shows variations of the previous data points with the tiers projected according to the rules above.

Let us briefly comment on the intuition behind these projection rules. We always want to project \( R \) since this is the only potential trigger for \( \text{nat}i \). For \([n]\), we do not want to project all instances as this may end up restricting the distribution of an \([n]\) that is not a suitable target anyways. In addition, we do not want to project \([n]\) itself as this would make it impossible to distinguish an \([n]\) that was projected as a potential \( \text{nat}i \)-target from one that was projected as an instance of \( C \). So instead, we project the sonorant after \([n]\). As a sonorant has no other reason to appear on the tier, it can act as an indirect representative of an \([n]\) that may be targeted by \( \text{nat}i \).

The left root boundary \( \sqrt{\cdot} \) matters only if it occurs between \( R \) and \([n]\). Since we build tiers from left-to-right, we cannot anticipate the presence of \([n]\) on the tier, and in the input string there is no upper bound on how far \([n]\) might be from \( \sqrt{\cdot} \). Hence we have to project \( \sqrt{\cdot} \) after every \( R \), even if \( R \) does not end up triggering \( \text{nat}i \). A redundant instance of \( \sqrt{\cdot} \) on the tier is no problem.

As for \( P \), its presence matters only when it occurs before a potential \( \text{nat}i \) target, so we project it only in these configurations. We also impose the requirement that the previous tier symbol is \( \sqrt{\cdot} \) as \( P \) needs a left root boundary between \( R \) and \([n]\) to become a blocker. This kind of mixing of input and output conditions is not necessary for \( P \), but it is essential for \( C \).

The coronal \( C \) is the strongest blocker. In contrast to \( \sqrt{\cdot} \), it does not depend on other material in the string, so it should be projected not only immediately after \( R \) but also if the previous tier symbol is \( \sqrt{\cdot} \) (from which we can infer that the tier symbol before that is \( R \)). A \( C \) between \([n]\) and a retroflex inhibits the latter’s ability to block \( \text{nat}i \), so we also need to project \( C \) if the previous tier symbol is \( S \) (our tier stand-in for \([n]\)). As arbitrary retroflex segments are projected only if the previous tier symbol is \( S \), projecting \( C \) after \( S \) effectively blocks projection of retroflexes. Note that we do not project \( C \) when it is an \([n]\) before
S as this is a potential nati-target and hence S is projected anyways.

4.2 SL grammar

Once the IOSL tier projection is in place, specifying the forbidden substrings is a simple task. Consider a segment [n] that is followed by S and hence a potential target for nati. If there are any R that can trigger nati, it suffices to consider the closest one. Given such a configuration \( R \cdots \cdot \cdot [n] \cdots \cdot \cdot S \), any tier will have the shape below:

\[
\cdots R (C | \sqrt{(C | P)}) S (C | S \mid \text{retroflex}) \cdots
\]

Here \((X | Y)\) means “X or Y or nothing”.

Based on this abstract template, the following substrings are illicit because they indicate an [n] in a configuration where nati should apply:

- \( RS \), and
- \( R \sqrt{S} X \) (where \( X \) is \( \kappa, C \), or S).

That these are the only four substrings that need to be forbidden is illustrated in Table 10. But note that \( \sqrt{\cdot} \) is always immediately preceded by R on the tier, so the illicit substrings for the second case can be shortened to \( \sqrt{S} \kappa, \sqrt{S} C \), and \( \sqrt{S} S \). Therefore the longest forbidden substring has at most 3 segments and the dependencies over the tier are SL-3. As a result, nati is IO-TSL-(3,2,3); these surprisingly low thresholds suggest that nati is still fairly simple from a formal perspective.

4.3 Removing abstract symbols

There are still a few minor points that merit addressing. As noted in §2.2, IO-TSL is not closed under relabeling, so the fact that the abstract patterns used in the previous sections are IO-TSL does not imply that nati itself is. This is the case only if one can compile out the abstract symbols R, S, C, P, and retroflex into specific segments like [n], [j], and [t]. No problems arise in cases where no segment corresponds to more than one abstract symbol. For example, [j] only matches S. If [j] were also a coronal blocker, then it would also match C and the account in the previous section would no longer work since it would not be clear if a [j] on a tier represents a blocker C or a sonorant S. In such a case, the use of abstract symbols would simplify the pattern in an illicit way. There are two instances of overlap in our analysis: R versus arbitrary retroflexes, and [n] belonging to both C and S.

Regarding the split between R and arbitrary retroflexes, it is actually unclear from the data whether the two are distinct classes. The available data includes only instances of segments matching R acting as blockers, though Ryan (2017) suggests based on his analysis that all retroflexes should be able to block. But even if the two are distinct, that is unproblematic for our account. The projection of R is less restricted than that of arbitrary retroflexes as the latter are only projected if the previous tier symbol is S. This discrepancy matters only in cases where a C-segment occurs after [n]. In this case, arbitrary reflexives do not project whereas R still does. Projecting an R-segment after a C has no consequences, though. If the preceding substring matches \( \sqrt{SC} \), then projecting R won’t salvage it. If it does not match this pattern, projecting R won’t make the tier ill-formed. Hence the minimal difference between R and arbitrary retroflexes — if it exists — is immaterial for our account.

That [n] belongs to both C and S is not much of an issue, either. Since S is projected only after [n], an [n] on the tier could be an instance of S only if there are [mn] clusters targeted by nati. In such cases, the entire cluster undergoes nati. For example, we see [nišaŋna], not *[nišaŋna] (cf. Table 2). There are two solutions to this. One option is to treat these not as [mn] clusters, but rather as a single segment [nː]. As long as the projection of S is generalized to be sensitive to both [n] and [nː], this data is handled correctly by our analysis. Alternatively, we could rely on the fact that [mn] clusters in our data are always followed by S. Hence we can limit S to vowels, glides, and [n] and still end up with a sonorant on the tier after each potential nati-target. Either way there are again safe ways to deal with the apparent overlap between the abstract symbols.

We find it interesting in connection to this that [j] is both sonorant and coronal but does not act as a coronal blocker. If it did, it would belong to both C and S, without an easy fix to rescue our analysis. Although Ryan (2017) accounts for the special status of [j] on the basis of its articulatory properties, the fact that its behavior is also predictable from a computational perspective is intriguing. Be that as it may, the important point is that the account in §4.1 and §4.2 captures nati even if the abstract symbols are compiled out to the actual segments.
as a long distance process, rather than local spreading of unpronounced material would
be an instance of feature coding, which destroys subregular complexity because every regular lan-
guage can be made SL-2 this way (see e.g. Rogers 1997). Ryan’s analysis may still be more appropri-
ate from a linguistic perspective, but for our pur-
poses it may incorrectly nullify the complexity of

4.4 Relevance and interpretation

Our analysis establishes IO-TSL as a tighter upper
bound on the complexity of nati when construed as
a single phonotactic constraint over strings. This
do not mean, though, that this is the only viable
view of nati, as it could have been analyzed as a
collection of individually simple constraints (cf.
Ryan, 2017), a condition on graph structures in the
sense of Jardine (2017), or a mapping from un-
derlying representations to surface forms. These
are all insightful perspectives, but they are orthog-
onal to our goal of bounding the overall com-
plexity of nati. Our finding that nati (and prob-
ably segmental phonology as a whole) is IO-TSL
is analogous to claims that syntax yields mildly
context-sensitive string languages (Joshi, 1985)
even though the actual representations are trees
with a tree-to-string mapping. The IO-TSL nature
of nati provides a rough complexity baseline on
which more nuanced and linguistically insightful
notions of complexity can be built.

We do find it interesting that IO-TSL as a natu-
geralization of TSL is sufficient to cap-
true nati. But IO-TSL is still too liberal an up-
ound bound. Just like the TSL-extensions used in
Baek (2017), De Santo and Graf (2017), and Yang
(2018), IO-TSL allows for unattested phenomena
such as first-last harmony (Lai, 2015). Future
work may identify subclasses of IO-TSL that al-
low for nati but not first-last harmony. IO-TSL in
its current form is nonetheless a fairly restrictive
upper bound on nati.

One final point of contention is our treatment of
nati as a long distance process, rather than local
retroflex spreading as suggested by Ryan (2017).
Unfortunately, there is no clear evidence that the
posted local spreading is visible in the output
string. Spreading of unpronounced material would

5 Conclusion

We have shown that even a highly complex pro-
cess like nati can be regarded as a local depen-
dency over tiers given a slightly more sophis-
ticated tier projection mechanism that considers
both the local context in the input and the preceed-
ing symbol(s) on the tier. This extension is natural
in the sense that it co-opts mechanisms that have
been independently proposed in the subregular lit-
erature as a more restricted model of rewrite rules.
Moreover, the complexity is fairly low as nati fits
into IO-TSL-(3, 2, 3). A careful reanalysis of the
data may be able to lower these thresholds even
more by incorporating independent restrictions on
the distribution of some segments. Allowing the
tier projection to proceed from right to left might
also affect complexity.

The effect of the increased power on learnability
is still unknown. IO-TSL-\((i, j, k)\) is a finite class
given upper bounds on \(i, j, \) and \(k\), which immedi-
ately entails that the class is learnable in the limit
from positive text (Gold, 1967). This leaves open
whether the class is efficiently learnable, as is the
case for TSL (Jardine and McMullin, 2017) and
the strictly local maps IO-TSL builds on (Chand-
dee et al., 2014, 2015). But IO-TSL adds two
serious complications: the learner does not have
access to the output of the tier projection function
in the training data, and inferring correct contexts
presupposes correctly built tiers.

<table>
<thead>
<tr>
<th>Example</th>
<th>Tier</th>
<th>Forbidden substring</th>
<th>Example</th>
<th>Tier</th>
<th>Forbidden substring</th>
</tr>
</thead>
<tbody>
<tr>
<td>nātēna</td>
<td>aŋ₁ ×</td>
<td>–</td>
<td>√mugná</td>
<td>p₁ ×</td>
<td>–</td>
</tr>
<tr>
<td>*nātēna</td>
<td>aŋ₂ ×</td>
<td>aŋ</td>
<td>*√mugná</td>
<td>t₁ ×</td>
<td>–</td>
</tr>
<tr>
<td>pñāţinaja</td>
<td>ñą ×</td>
<td>–</td>
<td>pñą√ñet₁</td>
<td>tפק ×</td>
<td>–</td>
</tr>
<tr>
<td>caanti</td>
<td>t₁ ×</td>
<td>–</td>
<td>*pñą√ñet₁</td>
<td>tפק ×</td>
<td>√et</td>
</tr>
<tr>
<td>utţa/ţhāna</td>
<td>ñą√a₁ ×</td>
<td>√a₁ ×</td>
<td>√bąźafanę́su</td>
<td>ñą×</td>
<td>–</td>
</tr>
<tr>
<td>*utţa/ţhāna</td>
<td>ñą√a₁ ×</td>
<td>√a₁ ×</td>
<td>*√bąźafanę́su</td>
<td>ñą×</td>
<td>–</td>
</tr>
<tr>
<td>pña/ţapnoati</td>
<td>tפק√pōt₁ ×</td>
<td>–</td>
<td>pña√nakš</td>
<td>tפק√aš₁×</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 10: Selected data points and ungrammatical variants from Tables 2 to 9 with their tiers and forbidden subsequences (if any); × is added to each tier for the sake of exposition.
Acknowledgments

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References


