# **The computational cost of generalizations:** *An example from micromorphology*

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#### Abstract

Morphotactics has been argued to be limited to the formal class of tier-based strictly local languages [1]. We claim that the level of the complexity of a pattern largely depends on the way it is morphologically analyzed. Using an example from adjectival inflection in Noon (Niger-Congo), we show that the complexity of this pattern falls in two different classes in the subregular hierarchy if viewed from different perspectives. The traditional segmentation of Noon affixes [8] yields a 3-TSL grammar, while the same pattern is 3-SSTSL under the perspective of micromorphology [9]. Both grammars require a locality window of 3 segments; however, the micromorphology-based analysis shows an increase in formal complexity, although it reduces the grammar size by defining complex affixes in terms of simpler ones.

# Subregular Languages

The formal class of regular languages can be decomposed in smaller subclasses that together form the *subregular hierarchy* [7, 5, i.a.].



# **Strictly Local**

Strictly local (SL) languages evaluate a string based on the *n*-grams it contains [6]. SL grammars capture local dependencies by blocking or allowing substrings of a certain length. As a result, it is not possible to capture a long-distance dependency with a SL grammar.

### SL language

Language L1: ab, abab, ababab, etc. Rules of language L1:

- well-formed words start with a:
- well-formed words end with b:
- the symbols a and b should alternate.

**Tier alphabet:**  $\Sigma = \{a, b\}$ **Negative SL grammar:**  $G_{NeqSL} = (* \rtimes b, *aa, *bb, *a\ltimes)$ 

# **Tier-based SL**

Tier-based strictly local (TSL) languages evaluate strings by looking for allowed or prohibited substrings while evaluating its tier [4]. Only the symbols present in the *tier alphabet* T are projected allowing the representation of long-distance dependencies locally over the tier.

TSL language

Language L2: b, aaab, aaba, baa Rules of language L2:

- *a* might be present or not;
- there must always be a single b.

**Tier alphabet:**  $T = \{b\}$ **Positive TSL grammar:**  $G_{PosTSL} = (\rtimes b, b \ltimes)$ 

<sup>ok</sup> abaa			*bab				
$\Join$		b			$\ltimes$	× b b ⊳	
×	a	b	a	a	ĸ	× b a b ⊳	

Figure 2: Examples of the TSL evaluation

# **Structure Sensitive TSL**

Structure sensitive tier-based strictly local grammars (SS-TSL) allow the projection of items on the tier only if they satisfy certain local condition(s) [3, 2]. SS-TSL languages encode long-distance dependencies that interact with local restrictions.



Figure 3: Examples of the SS-TSL evaluation

# **Noon Adjective Inflection**

The inflectional system of adjectives in Noon is very complex, but we restrict our focus on the attributive prefix and the definite suffix.

# **Traditional Approach**

The Noon attributive prefix and definite suffix are single morphemes regardless of their internal structures [8]. Both of these morphemes express the class of the dependent noun as well.

- (1) waas wi-yak road ATTR-big 'a big road (near you)'
- (2) waas-um wi-yak-um road-DEF ATTR-big-DEF 'the big road (near you)'



In this case, the class markers can appear only in two positions: immediately preceding the prefixal or the suffixal formation. Moreover, the class marker needs to be exactly the same in both positions.

# **Formal Analysis of Noon Pattern**

We assume the length of the stem to be potentially unbounded, and use the marker # in order to indicate the edges of the stem.

# **Micromorphological Perspective**

CM-PF-#-RT-#

(4) Indefinite adjectives (5) Definite adjectives CM-PF-#-RT-#-CM-SF

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Class	Indefinite	Definite
<b>C1</b>	wi-yak	wi-yak-wum
<b>C2</b>	fi-yak	fi-yak-fum

# Micromorphology

On the contrary, micromorphology states that a morphological unit may be morphologically complex, i.e. an affix may be a combination of other affixes [9]. According to this approach, the same adjective wiyakwum is segmented as shown in (3). (CM – class markers, PF – prefixal formative, SF - suffixal formative, RT - stem.

(3) w-i-yak-w-um CM-PF-RT-CM-SF 'the big (one)'

Class	Indefinite	Definite
<b>C1</b>	w-i-yak	w-i-yak-w-um
<b>C2</b>	f-i-yak	f-i-yak-f-um

Complexity of Noon pattern:

• Micromorphology: 3-SS-TSL  $\rightsquigarrow$  less restrictive  $\odot$ ; • **Traditional**: 3-TSL  $\rightsquigarrow$  more restrictive  $\bigcirc$ .

## Noon pattern: 3-SS-TSL analysis

 $T = \{ CM, PF^{after cm}, SF^{after cm}, \# \}$  $G_{SSTSL} = (\rtimes - CM - PF, CM - PF - \#, PF - \#, \# - \# - CM, \# - CM - SF,$  $CM-SF-\ltimes, \#-\#-\ltimes)$ 

<sup>ok</sup>CM-PF-#-RT-#-CM-SF  $\rtimes$  CM PF # # CM SF  $\ltimes$  $\rtimes$  CM PF # RT # CM SF  $\ltimes$ 

Figure 4: SS-TSL analysis of Noon adjectives

\*CM-X-PF-#-RT-# × CM # # × → CM X PF # RT # K

Figure 5: SS-TSL analysis of Noon adjectives [cont.]

# **Traditional Perspective**

Under the traditional perspective, the CM-PF and CM-SF sequences are single morphemes ATTR and DEF, respectively.

(6) Indefinit ATTR-#-

(7) Definite ATTR-#-

#### Noon pat

 $T = \{ATT\}$  $G_{TSL} = ($ 

# Conclusion

Noon inflectional morphotactic pattern varies in its computational complexity based on how it is analyzed.

We are not discriminating one morphotactic approach over the other. In-

stead, we show that the encoding of the formalism largely affects its computational complexity: it is not always the case that simplifying the way basic elements are represented reduces the overall complexity of the resulting system.

# References

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e adjectives RT-#
adjectives RT-#-DEF
tern: 3-TSL analysis
[¤-ATTR-#, ATTR-#-#, #-#-⋉, #-#-DEF, #-DEF-⋉)
$^{ok}$ ATTR-#-RT-#-DEF → ATTR # # DEF × → ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
Figure 6: TSL analysis of Noon adjectives

• Traditionally, it falls into the subregular class of TSL languages.

• Micromorphologically, the same pattern is computationally more complex, and needs a SS-TSL grammar in order to be captured.

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