CHAPTER 34

THE COMPUTATIONAL POWER OF HARMONIC FORMS

ALËNA AKSËNOVA, JONATHAN RAWSKI, THOMAS GRAF, AND JEFFREY HEINZ

34.1 Computation and phonology

It is hard to think of an area of science which has not been enriched by studying a subject computationally. At its core is the ability to formally define problems and processes and ask how (or whether) they can be solved computationally and, if so, with which resources. For phonology, it means asking about the nature of the phonological representations, constraints, and processes present in the phonological grammar. For phonologists, the big questions are: What constitutes knowledge of phonological well-formedness? What representations are used to determine that knowledge? Each of these questions can be stated computationally. What types of functions constitute phonotactics and phonological processes? What data structures are manipulated in these processes?

This chapter studies vowel harmony (VH) from this computational perspective. We primarily study necessary and sufficient conditions on the types of surface constraints present in VH *phonotactics*. The second case, the conditions on the types of constraints on harmony *as a process*, are focused on in Chapter 22, this volume. From a computational perspective, while there are many similarities between functions which output well-formedness values and functions which output structures, there are also some key differences.

The takeaway is that the computational complexity of grammars generating patterns obeying VH appears to fall within a well-defined class of finite-state, or regular, grammars (Johnson 1972; Kaplan and Kay 1994). This means the memory required to compute the output is bound by a constant no matter the size of the input. Furthermore, the computational conditions on VH can be even more strongly characterized using familiar phonological notions of tiers, which relativizes locality conditions (Goldsmith 1976a). This is in line with Heinz (2011), who argues that many phonological analyses can be made in terms of weaker, *sub-regular* classes.

34.2 Complexity of surface forms

This section focuses on classes of grammars which necessarily characterize local and long-distance dependencies by directly referencing surface structure, or projecting elements relevant for a certain process on a tier, therefore 'ignoring' all the intervening irrelevant material. These grammars evaluate surface forms. If a surface form violates some harmony constraint, then one should be able to write a grammar accounting for that fact. For VH tiers, typically it is some set of vowels that are projected. We show that, in the tier case, a single tier is always enough for vowel harmonies. When several harmonic processes occur within the same language, we refer to this as *double harmony*. For these cases, if more than a single-tier alphabet is needed, those tier alphabets can be either in a subset–superset relation, or completely incomparable. Interestingly, the case when tier alphabets are partially overlapping seems to be unattested.

In this section we overview the particular classes of languages and grammars which characterize harmony patterns. Readers are referred to Rogers and Pullum (2011) and Rogers et al. (2013) for more details on this class and related classes.

34.2.1 Strictly Local grammars

The core idea behind Strictly Local (SL) grammars is that well-formed strings are those which do not contain any *forbidden sub-strings*. The grammar is thus a finite list of these forbidden sub-strings.

Formally, a SL grammar consists of the set of k-grams (sub-strings of length k) G_{SL} that must not be contained within a well-formed string of the language.

For example, in Russian (Slavic), obstruents in a cluster must agree in voicing; see the data below in (1a–b).

(1a) [s]-tajat^j 'PREF-melt' *[z]-tajat^j
(1b) [z]-bit^j 'PREF-beat' *[s]-bit^j

The telic prefix *s-* is realized as [s] if followed by a voiceless consonant (1a), and as [z] if followed by a voiced one (1b). In this case, the alphabet $\Sigma = \{s, z, t, b, j, a, i\}$. The grammar prohibits combinations of obstruents with mixed voicing: $G_{SL} = \langle$*sz, *zs, *sb, *bs, *zt, *tz, *tb, *bt$\rangle$. The grammar then correctly rules out the forbidden forms of the words given above, where the prefix disagrees in voicing with the following obstruent.

As another example, consider VH in Lokaa (Niger-Congo). In this language, a non-high vowel agrees with the preceding non-high vowel in advanced tongue root (ATR). Furthermore, all high vowels and consonants are transparent for harmony. That surface forms exhibit this pattern is plausibly due to a harmony process exemplified by the rule in (2) that corresponds to Chomsky and Halle's *Sound pattern of English*.

(2)
$$\begin{bmatrix} -\text{cons} \\ -\text{high} \end{bmatrix} \rightarrow \begin{bmatrix} \alpha \text{ tense} \end{bmatrix} / \begin{bmatrix} -\text{consonant} \\ -\text{high} \\ \alpha \text{ tense} \end{bmatrix} \ldots \_$$

The examples below illustrating well-formed and ill-formed surface forms are from Akinlabi (2009).

(3)

a.	èsìsòn	'smoke'	*èsìsɔ̀n
b.	ɛ̀sísɔ̀n	'housefly'	*ɛ̀sísòn
c.	lèjìmə̀	'matriclan'	*lèjìmà
d.	ɛ́kílìkà	'kind of plant'	*ékílìkà

In this case, the agreeing items are not adjacent to each other. Strings such as (3a) or (3b) require 5-grams to capture this pattern, because there are three intervening elements between the two agreeing non-high vowels. But for (3d), even this window size is not enough: there are five segments between *ɛ* and *a*. Because there is no upper bound on the amount of material which separates the two non-high vowels agreeing with respect to the [tense] feature, no SL grammar can be constructed to capture this pattern (see arguments in Rogers and Pullum 2011).

34.2.2 Strictly piecewise grammars

While the harmony phenomenon in Lokaa cannot be described with a Strictly *k*-Local grammar for any *k*, the long-distance dependency does contain a measure of locality that can be seen by adjusting the structural information from *sub-strings*, which capture adjacent segments (*aa*), to *sub-sequences*, which characterize possibly non-adjacent segments (*a* … *a*) (Heinz 2010). In this case, collections of sub-sequences that are *k* segments long form a counterpart to the class of Strictly Local grammars, called the Strictly Piecewise (SP) grammars, which characterize Strictly Piecewise stringsets (Rogers et al. 2010). The particular set of length-2 sub-sequences that properly characterizes long-distance Lokaa harmony can be seen below in Table 34.1.

Table 34.1 SP grammar for Lokaa harmony

1.	*[α tense, –high] … [–α tense, –high]
	H_{ATR} = {*ɛ … e, *e … ɛ, *ɛ … o, *o … ɛ, *ɛ … ə, *ə … ɛ, *ɔ … e, *e … ɔ, *ɔ … ə, *ə … ɔ, *ɔ … o, *o … ɔ, *a … ə, *ə … a, *a … o, *o … a, *a … e, *e … a}

Heinz (2010) explains that SP grammars can account for harmonic forms without blocking effects but not harmonic forms with blocking effects. The tier-based grammars discussed next can account for both.

34.2.3 Tier-based Strictly Local grammars

Tier-based Strictly Local (TSL) grammars (Heinz et al. 2011) capture non-local dependencies by projecting elements of a certain type on a *tier* in order to achieve locality among the remote units.

A long-distance dependency can be viewed as local over the tier, because all the intervening material between the two dependent elements is ignored. As in SL grammars, forbidden sub-strings of segments can be ruled out over the tier. A TSL grammar consists of the tier alphabet T and the set of k-grams G_{TSL} that must not be found in a tier representation of a well-formed string.

Lokaa VH, described in the previous subsection, affected vowels that are arbitrarily far removed from the triggering vowel , and only those vowels. Consequently, as explained, it is not possible to construct an SL grammar capturing this process. However, the power of TSL languages allows us to project remote dependent items on a tier, and block illicit combinations over this tier.

Table 34.2 shows the active constraints on tier-adjacent non-high vowels, first as an illicit featural configuration, and second in terms of the corresponding illicit segmental bigrams, which would constitute the forbidden substrings in the TSL grammar H_{ATR}. In this case, the tier alphabet includes all non-high vowels presented in the language, and the pattern of ATR agreement observed in surface forms is obtained simply by blocking combinations of non-high vowels disagreeing in their [tense] specifications.

Table 34.2 TSL grammar for Lokaa harmony

	Tier of non-high vowels T = {ɛ, e, o, ə, ɔ, a}
2.	*[α tense] [$-\alpha$ tense]
	H_{ATR} = {*ɛe, *eɛ, *ɛo, *oɛ, *ɛə, *əɛ, *ɔe, *eɔ, *ɔə, *əɔ, *ɔo, *oɔ, *aə, *əa, *ao, *oa, *ae, *ea}

FIGURE 34.1 ATR harmony in Lokaa.

Figure 34.1 illustrates this analysis.

The left sub-figure shows the well-formed word *èsìsòn*. Only non-high vowels (*e* and *o*) are projected on the tier, and their combination *eo* is not among those that need to be ruled out, so the word *èsìsòn* is considered acceptable. Another sub-figure shows the ill-formed word **èsìsɔ̀n*, where the two non-high vowels *è* and *ɔ̀* disagree in their [tense] specifications. After *è* and *ɔ̀* are projected on a tier, we see that the bigram **eɔ* is banned by the grammar, and therefore this word must be ruled out.

34.2.4 Multiple Tier-based Strictly Local languages

In many languages, segments agree in more than just a single feature. For example, in Turkish (Turkic), vowels agree in backness and rounding; in Imdlawn Tashlhiyt (Afro-Asiatic), sibilants agree in voicing and anteriority, and in Bukusu (Atlantic-Congo), one agreement process involves vowels and another one involves liquids. The question is: *Are there any restrictions on the relationships among the tier alphabets in a language with more than one long-distance feature harmony?* There are four logical possible relations between distinct tier alphabets T_1 and T_2, as shown in Figure 34.2.

Firstly, T_1 can be *the same* as T_2—that is, the set of blockers and undergoers is the same for both harmonic processes (same tier). Jumping ahead, this seems to be always sufficient for double harmonies that only involve vowels. Alternatively, T_1 might *share no common elements* with T_2, as seen with independent vowel and consonant harmonies (disjoint tiers).

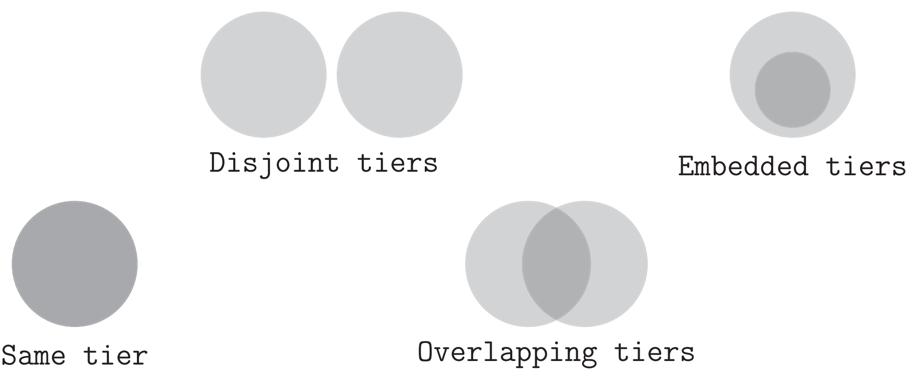

FIGURE 34.2 Possible tier relations.

T_1 can also be *a subset of* T_2, and it seems to be the case for some of the consonant harmonies (embedded tiers). However, the case when T_1 only *partially overlaps* with T_2 seems to be typologically unattested (overlapping tiers).

34.3 Attested patterns

In this section, we use the previously discussed grammar types to capture different harmonic patterns. A harmonic process picks out a set of elements (*undergoers*) and establishes an agreement relation among them with respect to a certain feature. This agreement might be blocked; there are some items (*blockers*) that can stop this spreading, so the previous harmonizing element will not affect the following one if a blocker is found between them. All other segments are *transparent* for the long-distance assimilation—they do not affect, and are unaffected by, the harmonic process. These terms are useful even when discussing static phonotactic patterns, even though we introduced them in the context of *processes*, and they are often used in that context as well.

A typological study of double VH patterns shows that they require just a single tier, even though more than one feature is being transmitted. For sibilant harmonies, one tier is not enough. Two tiers need to be projected, and their tier alphabets are in a set–subset relation. Languages with both vowel and consonantal harmonies also require two tiers. However, those tiers do not share any common element in their tier alphabets—that is, they are disjoint. So far, there are no attested cases of intersecting alphabets, where the sets of items involved in the two kinds of processes only partially overlap. Out of the logical possibilities, the partially overlapping case is the most numerous!

34.3.1 Double vowel harmonies

The naïve expectation could be that each of the harmonic features of the double harmony will require its own tier. However, the data show that, with double VH, both harmonies fit on the same tier. This does not mean that undergoers and blockers are the same for both harmonies, just that none of the items taking part in one harmony is irrelevant for the other one. In most cases, one of the harmonies affects all vowels, and the other spreads its feature only among the tier-adjacent [α high] vowels, whereas [$-\alpha$ high] ones function as blockers (e.g., Turkish, Mongolian, Buryat, Tatar). Sometimes two features are transmitted simultaneously (e.g., Kirghiz). In other cases, all vowels function as undergoers with respect to both harmonies, while only a subset of those vowels can further spread both harmonic features—the others function as harmonizing blockers (e.g., Yakut). See Kaun (1995) for numerous examples of labial feature spreading that commonly serve as one of the two harmonies in a double-harmonic system. This makes it possible to formulate two main requirements for the abstract harmonies A and B in order to fit them on the same tier:

- The set of items transparent for the harmonies A and B must be identical.
- The blockers and undergoers for the harmony A must play some role in the harmonic process B—that is, they must be either blockers or undergoers.

Here we show three examples of such double spreading: fronting and rounding harmony in Kirghiz, where two features are transmitted together—that is, the set of undergoers is the same for both kinds of spreading; ATR and rounding harmony in Buryat, where some undergoers of the ATR harmony function as blockers of the labial one; and fronting and labial harmony in Yakut, where some harmonizing items block the spreading in certain configurations.

34.3.1.1 Kirghiz (Turkic)

Double VH in Kirghiz spreads the features of frontness and rounding simultaneously: All vowels within a word must agree in these two features. This would be a consequence of the following rule:

(4)
$$\begin{bmatrix}-\text{ cons}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{front}\\ \beta\,\text{round}\end{bmatrix} \ / \ \begin{bmatrix}-\ \text{cons}\\ \alpha\ \text{front}\\ \beta\,\text{round}\end{bmatrix}\ldots\_$$

A locative affix with a non-high vowel (*-de, -dö, -da, -to*) and a genitive affix with a high vowel (*-nin, -din, -dün, -tun*) are used in the examples (5a–f) below from Nanáyev (1950). Here, we present data from a dialect of Kirghiz (Nanayev 1950). See Herbert and Poppe (1963), Kaun (1995), and Washington (chapter 59, this volume) for additional discussion of Kirghiz and its dialects, including whether such a dialect exists.

(5)

a.	ot-to	'fire-LOC'	d.	ot-tun	'fire-GEN'
b.	kim-de	'who-LOC'	e.	kim-din	'who-GEN'
c.	uj-do	'house-LOC'	f.	uj-dun	'house-GEN'

Using two binary harmonizing features, all vowels within a word have a choice among the following four possibilities: [+front, +round], [+front, –round], [–front, +round], and [–front, –round]. For each option, there is an example in the data above that shows that affixes agree with the root in these two parameters. For instance, the fronted rounded vowel in *üj* 'house' spreads its feature specifications both to non-high (5c) and high (5f) affixes. Provided the generalizations above are accurate, Table 34.3 shows the feature configurations that must be avoided in the well-formed vowel sequence of Kirghiz words: any bigram where the two vowels disagree in fronting or rounding must be blocked.

Table 34.3 TSL grammar for Kirghiz harmony

	Vowel tier T = {a, ɨ, e, i, o, ö, u, ü}
1.	*[α front] [β front]
	H_{front} = {*ai, *ae, *aö, *aü, *oi, *oe, *oö, *oü, *ɨi, *ɨe, *ɨö, *ɨü, *ui, **ue, *uö, *uü, *ia, *io, *iɨ, *iu, *ea, *eo, *eɨ, *eu, *öa, *öo, *öɨ, *öu, **üa, *üo, *üɨ, *üu}
2.	*[α round] [β round]
	H_{round} = {*oa, *oɨ, *ua, *uɨ, *öi, *öe, *üi, *üe, *ao, *au, *iö, *iü, **ɨo, *ɨu, *eö, *eü, *oe, *oi, *öa, *öɨ, *ue, *ui, *üa, *üɨ, *eo, *io, **aö, *ɨö, *eu, *iu, *aü, *ɨü}

Both spreading types operate over the same set of segments, so the tier alphabet T consists of all the vowels. To enforce frontness harmony, combinations of vowels disagreeing in [front] specification must be blocked by H_{front}. In addition, sub-strings with vowels of different [round] specifications also need to be ruled out by H_{round}. Note that only one TSL grammar is needed to capture this pattern. Its tier alphabet is T, and the grammar G_{TSL} is a combination of the H_{front} and H_{round} sets of banned bigrams.

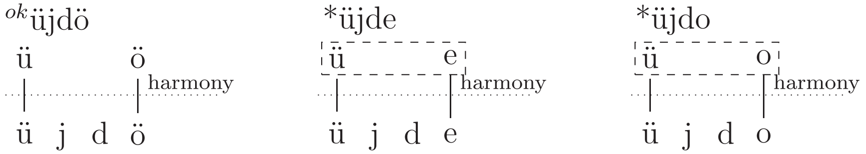

FIGURE 34.3 Frontness and labial harmony in Kirghiz.

Consider Figure 34.3 as the graphic example of the grammar constructed above in action. The left sub-figure represents the well-formed word *üjdö*. Both vowels are projected on a tier, and nothing is blocked, because the combination *üö* is permitted: Both *ü* and *ö* agree in their fronting and rounding features. The middle sub-figure shows a violation of the rounding harmony: Bigram **üe* is ruled out by the H_{round} part of the grammar. Likewise, the fronting harmony violation (see the right sub-figure) is ruled out by H_{front}, because **üo* is listed as an ill-formed sub-string. In Kirghiz, the set of undergoers for one harmonic spreading is exactly the same as for the other one, so, as a result, only one TSL grammar is needed.

34.3.1.2 Buryat (Mongolian)

In Buryat, all vowels within a word must agree in ATR. All tier-adjacent non-high vowels agree in rounding, unless there is an intervening high vowel that blocks this assimilation. As a result, the only possible positions in which rounded non-high vowels can be found are initial syllables, and other positions if they are licensed by labial spreading (Poppe 1960). The set of transparent items is the same for both harmonies, and includes the only non-rounded high vowel /i/ and all consonants (see van der Hulst and Smith 1987; Skribnik 2003; Svantesson et al. 2005). This set of well-formed surface forms can be understood as the consequence of the SPE-style rules (6)–(8).

(6) $$\begin{bmatrix}-\text{cons}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{tense}\end{bmatrix} \ / \begin{bmatrix}-\ \text{cons}\\ \alpha\ \text{tense}\end{bmatrix} \cdots \text{or /i/ } -$$

(7) $$\begin{bmatrix}-\text{cons}\\ -\text{high}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{round}\end{bmatrix} / \begin{bmatrix}-\ \ \text{cons}\\ -\ \ \text{high}\\ \alpha\ \text{round}\end{bmatrix} \cdots \text{or /i/ } -$$

(8) $$\begin{bmatrix}-\text{cons}\\ -\text{high}\end{bmatrix} \rightarrow \begin{bmatrix}-\text{round}\end{bmatrix} / \begin{bmatrix}-\text{cons}\\ +\text{high}\end{bmatrix} \cdots \text{or /i/ } -$$

In the examples below, the harmony process is illustrated using the following two affixes: the causative suffix *-ʊ:l, -u:l*, where the vowel is specified as high, that agrees with the stem only in ATR; the perfective affix *-a:d, -ɔ:d, -e:d, -o:d* with a non-high vowel that agrees with the preceding segment in ATR and, if that segment is also non-high, in rounding.

In (9a), the non-high perfective affix agrees with the non-high root vowel in ATR and rounding: both vowels are lax and rounded. Adding the high causative affix between them, as in (9b), results in blocking of labial spreading: The perfective affix no longer agrees with the stem in rounding, because they are separated from each other by an intervening high vowel. Examples (9c–d) show the same effect for the tense root. Examples (9e) and (9f) show the transparency of the intervening vowel /i/.

(9)

a.	ɔr-ɔ:d	‘enter-PERF’	*ɔr-a:d
b.	ɔr-ʊ:l-a:d	‘enter-CAUS-PERF’	*ɔr-ʊ:l-ɔ:d
c.	to:r-o:d	‘wander-PERF’	*to:r-e:d

d.	to:r-u:l-e:d	'wander-CAUS-PERF'	*to:r-u:l-o:d
e.	mɔrin-ɔ:	'horse-POSS'	*mɔrin-a:
f.	o:rin-go:	'group-POSS'	*o:rin-ge:

Table 34.4 factors the TSL grammar for Buryat into its major parts. The set of elements involved in each of these two kinds of spreading is the same, so the tier alphabet T consists of all relevant vowels for harmony. H_{ATR} blocks combinations of vowels that disagree in tenseness, and $H_{r1} \cup H_{r2}$ enforce the tier-adjacent non-high vowels to harmonize in rounding, and also block rounded non-high vowels after the high ones. Again, only one TSL grammar is needed: its tier alphabet is T, and $G_{TSL} = H_{ATR} \cup H_{r1} \cup H_{r2}$.

Table 34.4 TSL grammar for Buryat harmony

Vowel tier (except /i/) T = {a, e, ɔ, o, ʊ, u}	
1.	*[α tense] [β tense]
	H_{ATR} = {*ɔo, *oɔ, *ʊu, *uʊ, *ae, *ea, *ʊo, *oʊ, *ʊe, *eʊ, *ɔu, *uɔ, **ɔe, *eɔ, *oa, *ao, *au, *ua}
2.	*[– high, α round] [– high, β round]
	H_{r1} = {*ɔa, *aɔ, *eo, *oe, *ao, *oa, *eɔ, *ɔe}
3.	*[+ high] [– high, + round]
	H_{r2} = {*ʊɔ, *uo, *uɔ, *ʊo}

In Table 34.4, the first line shows that the tier-adjacent non-high vowels must agree in rounding, so the tier bigram *oo* is allowed, whereas illicit sub-strings such as **oe* are ruled out by the grammar H_{r1}. But if there is an intervening high vowel between them, as on the second line, labial harmony does not happen: occurrence of the rounded non-high vowel after the high one (**uo*) is also blocked by the grammar. As this analysis shows, Buryat double VH also requires just a single tier, because both instances of spreading operate over the same alphabet. This is shown in Figure 34.4.

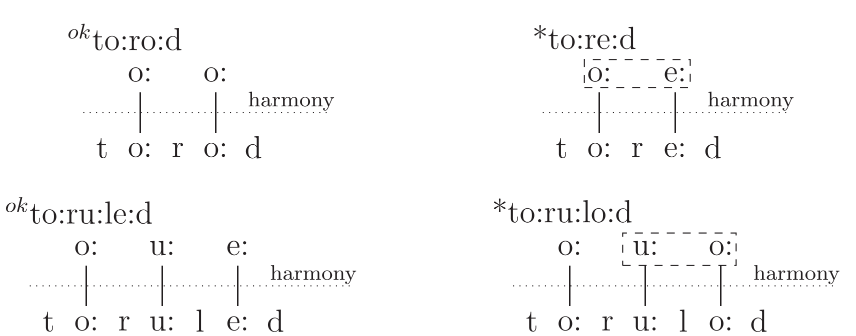

FIGURE 34.4 ATR and labial harmony in Buryat.

34.3.1.3 Yakut (Turkic)

In Yakut, all vowels must agree in fronting. Labial harmony spreads from the low vowels to both low and high ones, and from the high vowels to the high ones, but it cannot spread from the high vowels to the low ones. The latter ones in this case function as *harmonizing blockers*: They harmonize with any previous vowel, but block the rounding assimilation in [+high][–high] configuration. See Sasa (2001, 2009) for details and an OT account. This pattern is the consequence of the rules outlined in examples (10)–(13).

(10) $$\begin{bmatrix}-\text{cons}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{front}\end{bmatrix} \;/\; \begin{bmatrix}-\ \text{cons}\\ \alpha\ \text{front}\end{bmatrix}\cdots\_\_$$

(11) $$\begin{bmatrix}-\text{cons}\\ +\text{high}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{round}\end{bmatrix} \;/\; \begin{bmatrix}-\ \ \text{cons}\\ \alpha\ \text{round}\end{bmatrix}\cdots\_$$

(12) $$\begin{bmatrix}-\text{cons}\\ -\text{high}\end{bmatrix} \rightarrow \begin{bmatrix}\alpha\ \text{round}\end{bmatrix} \;/\; \begin{bmatrix}-\ \ \text{cons}\\ -\ \ \text{high}\\ \alpha\ \text{round}\end{bmatrix}\cdots\_$$

(13) $$\begin{bmatrix}-\text{cons}\\ -\text{high}\end{bmatrix} \rightarrow \begin{bmatrix}-\text{round}\end{bmatrix} \;/\; \begin{bmatrix}-\text{cons}\\ +\text{high}\end{bmatrix}\ldots\_$$

The accusative affix *-(n)ü, -(n)u, -(n)ɨ, -(n)i* with a high vowel and the plural marker *-lor, -lör, -lar, -ler* with a non-high vowel demonstrate this pattern; see examples (14a–h) below from Kaun (1995).

(14)

a.	oɣo-lor	‘child-PL’	*oɣo-lar
b.	börö-lör	‘wolf-PL’	*börö-ler
c.	oɣo-nu	‘child-ACC’	*oɣo-nɨ
d.	börö-nü	‘wolf-ACC’	*börö-ni
e.	murum-u	‘nose-ACC’	*murum-ɨ
f.	tünnük-ü	‘window-ACC’	*tünnük-i
g.	ojum-lar	‘shaman-PL’	*ojum-lor
h.	tünnük-ler	‘window-PL’	*tünnük-lör

High suffixal vowels agree with any preceding vowel in both fronting and rounding (14c–f), whereas low vowels always agree in fronting, and agree in rounding only when the preceding vowel is non-high (14a–b); otherwise, they are unrounded (14g–h).

Table 34.5 factors the TSL grammar for Yakut into its major parts. The tier alphabet T consists of all Yakut vowels. H_{front} rules out sequences of vowels that disagree in fronting, whereas $H_{r1} \cup H_{r2} \cup H_{r3}$ blocks occurrence of a rounded low vowel if it is preceded by a

Table 34.5 TSL grammar for Yakut harmony

Vowel tier T = {a, ɨ, e, i, o, ö, u, ü}	
1.	*[α front] [$-\alpha$ front]
	H_{front} = {*ai, *ae, *aö, *aü, *oi, *oe, *oö, *oü, *ɨi, *ɨe, *ɨö, *ɨü, *ui, **ue, *uö, *uü, *ia, *io, *iɨ, *iu, *ea, *eo, *eɨ, *eu, *öa, *öo, *öɨ, *öu, **üa, *üo, *üɨ, *üu}
2.	*[+ high, α round] [+ high, $-\alpha$ round]
	H_{r1}= {*uɨ, *üi, *iü, *ɨu, *iu, *ɨü, *ui, *üɨ}
3.	*[+ high] [– high, + round]
	H_{r2}= {*üö, *uo, *iö, *ɨo, *io, *ɨö, *uö, *üö}
4.	*[– high, α round] [$-\alpha$ round]
	H_{r3}= {*oa, *oɨ, *öi, *öe, *ao, *au,*eö, *eü, *aö, *aü, *eo, *eu, *oi, **oe, *öa, *öɨ}

high one, and also any other combination of vowels that disagree in their labial features. The obtained TSL grammar operates over the tier alphabet T and its list of illicit substrings is $G_{TSL} = H_{front} \cup H_{r1} \cup H_{r2} \cup H_{r3}$.

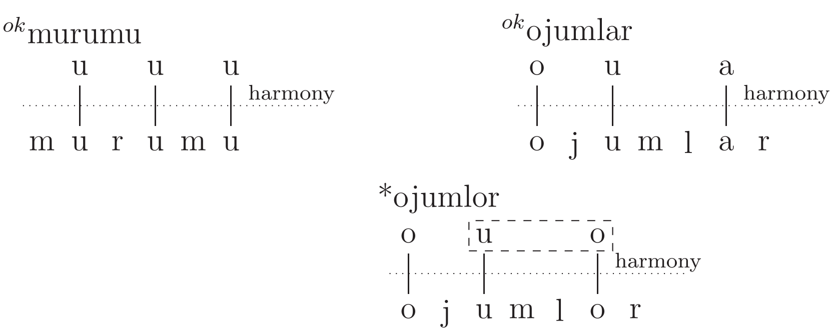

FIGURE 34.5 Fronting and labial harmony in Yakut.

Figure 34.5 shows that under this analysis, the word *murumu* is well formed—the labial harmony spreads among high vowels. It also spreads from the non-high vowel *o* to the following high vowel *u* in the word *ojumlar*. However, it cannot spread from a high vowel to a low one, so **ojumlor* is blocked as the illicit bigram **uo* is found on its vowel tier. As in the previous cases, a single tier is enough to capture Yakut double VH.

34.3.2 Double vowel harmonies: Summary

In the previous three subsections, we have shown three examples of double vowel harmonies where only one TSL grammar was needed in order to capture the pattern. It was possible because neither of the vowels taking part in one spreading was irrelevant for the other harmony.

1. **Kirghiz:** All vowels within a word harmonize in both [front] and [round] features, so all vowels are undergoers for both instances of spreading.
2. **Buryat:** All vowels within a word harmonize in [tense], and all non-high vowels agree with respect to the [round] feature, whereas intervening high vowels block this spreading. Thus, all vowels are undergoers of [tense] harmony, additionally, non-high vowels are undergoers of the labial one, and the high vowels function as blockers in the latter case.
3. **Yakut:** All vowels harmonize in [front] and [round], except for the configuration when a high vowel precedes a non-high one: In this case, labial spreading is blocked, whereas fronting harmony is unaffected.

34.3.3 Vowel harmony and consonant harmony

Previously, we showed double harmonies only among vowels or only among consonants. Here, the two harmonies target different sets of elements—the first is consonantal harmony and the second is vowel assimilation.

34.3.3.1 Kikongo (Bantu)

There are both consonant and vowel harmonies in Kikongo. VH enforces vowel agreement in height, whereas nasal agreement turns /d/ and /l/ into /n/ if preceded by a nasal in the stem (see Ao 1991; Hyman 1998; and the rule in (15)). The data below is adopted from Hyman (1998). In these examples, the applicative suffix *-el, -il* and the reversive transitive suffix *-ol, -ul* illustrate VH in height.

(15)
$$\left[-\text{cons}\right] \rightarrow \left[\alpha\,\text{high}\right] \;/\; \begin{bmatrix} -\,\text{cons} \\ \alpha\,\text{high} \end{bmatrix} \cdots \_$$

(16)

a.	-somp-el-	'attach-APPL'	(e)	-tomb-ol-	'do-TRANS'
b.	-leng-el-	'languish-APPL'	(f)	-lemb-ol-	'broom-TRANS'
c.	-sik-il-	'support-APPL'	(g)	-vil-ul-	'move-TRANS'
d.	-vur-il-	'surpass-APPL'	(h)	-bub-ul-	'bribe-TRANS'

In this language, suffixes are specified for rounding, and acquire height specification depending on the stem vowel. In (16a–b) and (16e–f), both vowels in the stem and in the affix are non-high, whereas (16c–d) and (16g–h) contain the high ones.

Table 34.6 TSL grammar for Kikongo vowel harmony

Vowel tier $T_v = \{e, o, i, u\}$	
1.	*[α hi] [β hi]
	H_v = {*oi, *ou, *ei, *eu, *io, *ie, *uo, *ue}

As shown in Table 34.6, this harmony operates over the tier of vowels T_v, and the grammar must rule out all combinations of vowels that disagree in height. Alongside VH, this language also has nasal consonant harmony. Segments /d/ and /l/ in the affix both become /n/ if nasal consonants /m/ or /n/ are present in a root. See examples below from Ao (1991), where *-idi* is the perfective active suffix and *-ulu* is its passive counterpart.

(17) $/d/, /l/ \rightarrow [n] / [m], [n] -$

(18)

a. -suk-idi- 'wash-PERF.ACT'

b. -nik-ini- 'ground-PERF.ACT'

c. -meng-ene- 'hate-PERF.ACT'

d. -suk-ulu- 'wash-PERF.PASS'

e. -nik-unu- 'ground-PERF.PASS'

f. -meng-ono- 'hate-PERF.PASS'

In (18a) and (18d), there are no nasals in a root, so the consonant in the affix is unchanged—it remains /d/ and /l/, respectively. However, when there are nasals /n/ or /m/ in the stem, both affixal /d/ and /l/ change to /n/: see (18b,e) and (18c,f), respectively.

Table 34.7 TSL grammar for Kikongo consonant harmony

T_n = {n, m, d, l}	
1.	*d [+nasal], *l [+nasal]
	H_n = {*nd, *nl, *md, *ml}

As Table 34.7 shows, only /d/, /l/, and nasals are involved in the process, so are projected over the tier, and the grammar H_n blocks occurrence of /d/ and /l/ after nasals. The two tier-based SL grammars have absolutely different tier alphabets, T_v and T_n, and cannot be combined, because nasals can occur between vowels, as well as vowels between nasals.

Figure 34.6 illustrates such an analysis for the Kikongo pattern. Two tiers are necessary to describe this pattern, because only these can provide the locality relations among vowels needed for VH, and /d/, /l/, and nasals for nasal assimilation. The well-formed word *nikunu* is permitted because its vowel tier string *iuu* does not violate the VH rule, and the nasal tier *nn* also satisfies the nasal harmony. Ill-formed combinations of segments like **io*, **uo*, and **nl* are ruled out by the corresponding grammars. Note that the two vowels /i/ and /u/ intervene between /n/ and /l/ in the rightmost sub-figure, and only the existence of the two separate tiers creates the locality needed for the TSL analysis of this pattern.

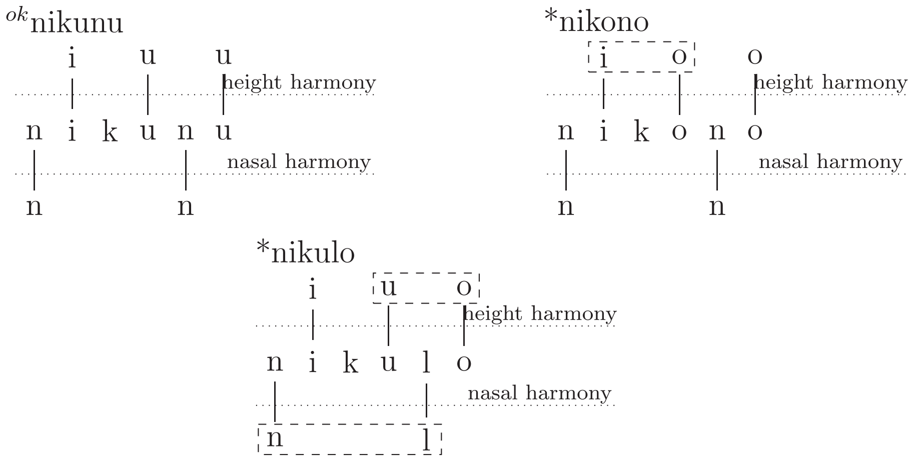

FIGURE 34.6 Vowel and nasal harmonies in Kikongo.

34.3.4 Typological summary

Table 34.8 summarizes some attested harmonic patterns where dependencies exist among more than one feature. The table briefly describes the surface pattern, and also its type in terms of the tier alphabet(s). **S** stands for a single tier—only one TSL grammar is required for both dependencies. **E** stands for embedding—that is, one tier alphabet is a subset of the other. **D** means the two tier alphabets are disjoint.

To our knowledge, there are no examples of the fourth possible type, where two tiers partially overlap (**P**). As Aksënova and Deshmukh (2018) show, if this absence was formulated as a linguistic hypothesis on phonologically possible tiers, it would drastically reduce the predicted tier combinations. They write, “For a set of 10 elements, this limitation excludes 95% of all possible tier alphabet organizations” (64). In other words, the attested types {**S, E, D**} constitute only a small fraction of the four logically possible types {**S, E, D, P**}.

34.4 Conclusion

The computational analysis of harmonic forms today bears much fruit. These insights and issues surrounding the properties of harmony define necessary and sufficient conditions on the class of functions characterizing the phonological knowledge to compute it. Further work in this area will continue to shed light on the nature of phonology, and long-distance dependencies such as those found in harmony will undoubtedly continue to play an important role.

Acknowledgments

We thank Adam McCollum and two reviewers for their helpful comments on an earlier draft.

Table 34.8 A summary of the reviewed phonotactic patterns with multiple long-distance dependencies

Type	Language	Description	References
S	Kirghiz	All vowels agree in both fronting and rounding.	Kaun (1995)
S	Kachin Khakass	All vowels agree in fronting. Tier-adjacent high vowels agree in rounding.	Korn (1969)
S	Yakut	All vowels agree in fronting. High vowels agree with any preceding vowel, non-high ones agree only with preceding non-high vowel.	Sasa (2001)
S	Karachay, Azerbaijani, Tuvan, Uyghur	All vowels agree in fronting. High vowels agree with any preceding vowel in rounding.	Kaun (1995) Comrie (1981a) Krueger (1977)
S	Kazakh	All vowels agree in fronting. In fronted words, all vowels agree in rounding. In non-fronted ones, only high vowels agree with the preceding vowel in rounding.	Korn (1969)
S	Shor	All vowels agree in fronting. In fronted words, all vowels agree in rounding. In non-fronted ones, tier-adjacent vowels agree in rounding only if they agree in height.	Korn (1969)
S	Buryat, Mongolian	All vowels agree in ATR. Non-high vowels agree in rounding, high vowels block it. Transparent element for both harmonies is /i/.	Poppe (1960) Svantesson et al. (2005)
S	Even, Oroch, Ulcha	All vowels agree in retracted tongue root (RTR). Tier-adjacent non-high vowels agree in rounding.	Comrie (1981a)
S	Tuareg	All sibilants agree both in voicing and anteriority.	Hansson (2010b)
E	Imdlawn Tashlhiyt	Sibilants agree in voicing and anteriority. Voiceless obstruents block the voicing harmony and are transparent for the anteriority harmony.	Hansson (2010a)
D	Kikongo, Kiyaka	Vowels agree in height. Consonants /d/ and /l/ assimilate to /n/ if preceded by a nasal.	Ao (1991) Hyman (1998)
D	Bukusu	Vowels agree in height. Liquid /l/ assimilates to /r/ if preceded by /r/.	Odden (1994)