

# An Alternate View on Strong Lexicalization in TAG

**Aniello De Santo, Alëna Aksënova and Thomas Graf**

Stony Brook University  
Department of Linguistics  
aniello.desanto@stonybrook.edu

Düsseldorf, June 29 - July 1 2016  
TAG+12

## A well-known fact

Lexicalized grammars are *good* for parsing algorithms

## Problem

TAGs not closed under strong lexicalization

## Idea

- generalize TAGs to multi-dimensional TAGs;
- lexicalization via increase in dimensionality:  
⇒ every  $d$ -TAG is strongly lexicalized by some  $(d+1)$ -TAG

## A well-known fact

Lexicalized grammars are *good* for parsing algorithms

## Problem

TAGs not closed under strong lexicalization

## Idea

- generalize TAGs to multi-dimensional TAGs;
- lexicalization via increase in dimensionality:  
⇒ every  $d$ -TAG is strongly lexicalized by some  $(d+1)$ -TAG

## A well-known fact

Lexicalized grammars are *good* for parsing algorithms

## Problem

TAGs not closed under strong lexicalization

## Idea

- generalize TAGs to multi-dimensional TAGs;
- lexicalization via increase in dimensionality:  
⇒ every  $d$ -TAG is strongly lexicalized by some  $(d+1)$ -TAG

- 1 Introduction
  - Lexicalization
  - Existing Results
- 2 Preliminaries
  - Adjunction & Substitution
  - TAGs as 3-d trees
  - TAGs as multi-dimensional structures
- 3 Strong Lexicalization
  - $d$ -TAGs are  $(d+1)$ -TSGs
  - $d$ -TAGs strongly lexicalize  $d$ -TSGs
- 4 Conclusion

A grammar is lexicalized if the atoms from which compound structures are assembled each contain a pronounced lexical item.

Lexicalized grammars are *finitely ambiguous*:

- recognition is decidable;
- parsing is simplified [Schabes et al., 1988]

An Essential Distinction

weak lexicalization vs strong lexicalization

A grammar is lexicalized if the atoms from which compound structures are assembled each contain a pronounced lexical item.

Lexicalized grammars are *finitely ambiguous*:

- recognition is decidable;
- parsing is simplified [Schabes et al., 1988]

## An Essential Distinction

weak lexicalization vs strong lexicalization

## Existing Results

- TAGs can be weakly lexicalized [Fujiyoshi, 2004]
- TAGs are not closed under strong lexicalization [Kuhlmann and Satta, 2012]
- TAGs are strongly lexicalized by context-free tree grammars of rank 2 [Maletti and Engelfriet, 2012]

## Aim of this Paper

Derive lexicalization properties of TAGs by generalizing to multidimensional structures

- Every  $d$ -dimensional TAG is a  $(d + 1)$ - dimensional TSG
- Every  $d$ -dimensional TSG is strongly lexicalized by some  $d$ -dimensional TAG
- $(d + 1)$ - TAGs strongly lexicalize  $d$ -TAGs



## Existing Results

- TAGs can be weakly lexicalized [Fujiyoshi, 2004]
- TAGs are not closed under strong lexicalization [Kuhlmann and Satta, 2012]
- TAGs are strongly lexicalized by context-free tree grammars of rank 2 [Maletti and Engelfriet, 2012]

## Aim of this Paper

Derive lexicalization properties of TAGs by generalizing to multidimensional structures

- Every  $d$ -dimensional TAG is a  $(d + 1)$ - dimensional TSG
- Every  $d$ -dimensional TSG is strongly lexicalized by some  $d$ -dimensional TAG
- $(d + 1)$ - TAGs strongly lexicalize  $d$ -TAGs

## Existing Results

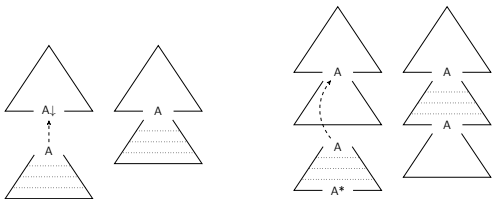
- TAGs can be weakly lexicalized [Fujiyoshi, 2004]
- TAGs are not closed under strong lexicalization [Kuhlmann and Satta, 2012]
- TAGs are strongly lexicalized by context-free tree grammars of rank 2 [Maletti and Engelfriet, 2012]

## Aim of this Paper

Derive lexicalization properties of TAGs by generalizing to multidimensional structures

- Every  $d$ -dimensional TAG is a  $(d + 1)$ - dimensional TSG
- Every  $d$ -dimensional TSG is strongly lexicalized by some  $d$ -dimensional TAG
- $(d + 1)$ - TAGs strongly lexicalize  $d$ -TAGs

# Adjunction & Substitution

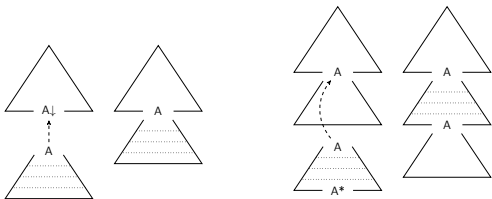


Substitution can be regarded as adjunction of a **footless** tree at a leaf node

## Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite leaf nodes

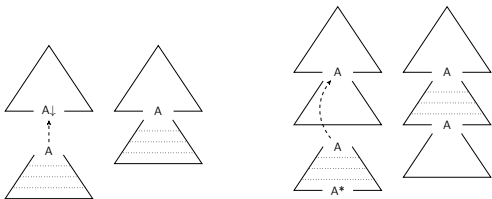
# Adjunction & Substitution



Substitution can be regarded as adjunction of a **footless** tree at a leaf node

Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite leaf nodes

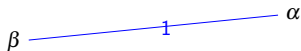
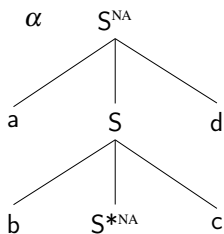
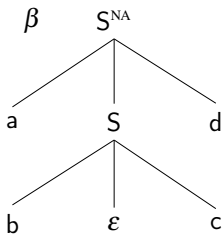


Substitution can be regarded as adjunction of a **footless** tree at a leaf node

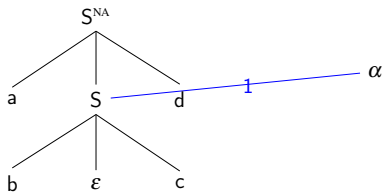
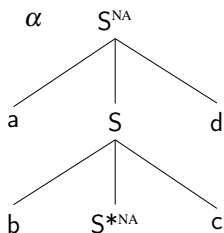
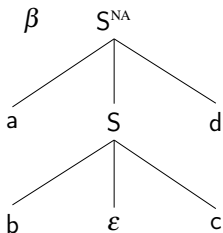
## Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite leaf nodes

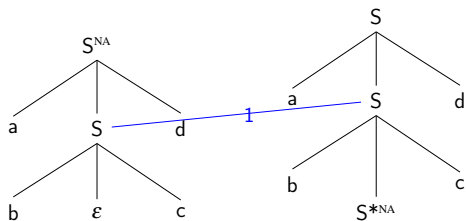
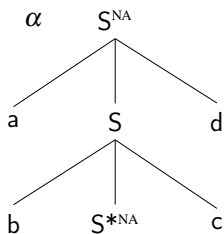
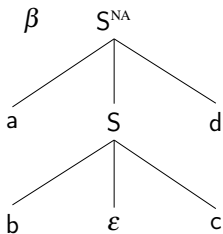
# TAGs as 3-d trees [Rogers, 1998]



# TAGs as 3-d trees [Rogers, 1998]

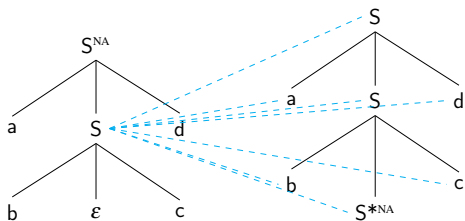
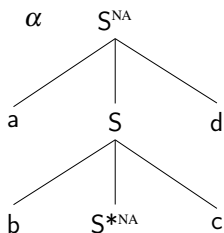
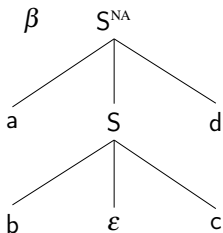


# TAGs as 3-d trees [Rogers, 1998]

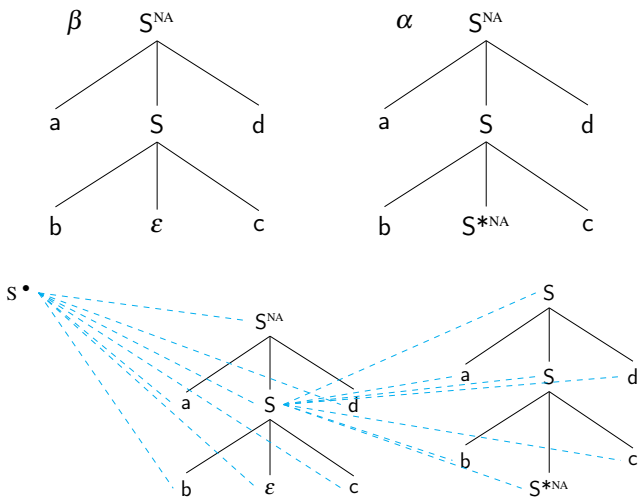




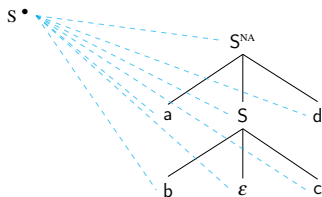
# TAGs as 3-d trees [Rogers, 1998]



# TAGs as 3-d trees [Rogers, 1998]



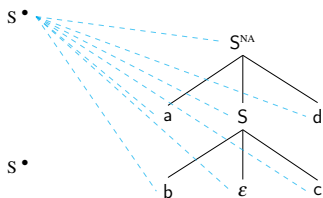
We can increase the dimensionality of a grammar



d-dimensional Local Structure

- d-dimensional mother
- $y^d$ : (d)-yield

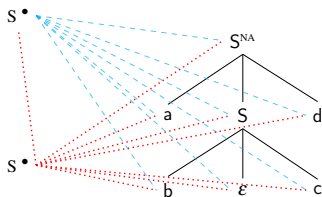
We can increase the dimensionality of a grammar



d-dimensional Local Structure

- d-dimensional mother
- $y^d$ : (d)-yield

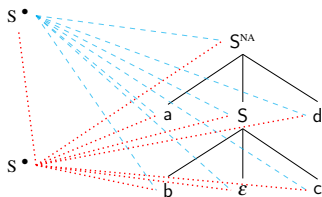
We can increase the dimensionality of a grammar



d-dimensional Local Structure

- d-dimensional mother
- $y^d$ : (d)-yield

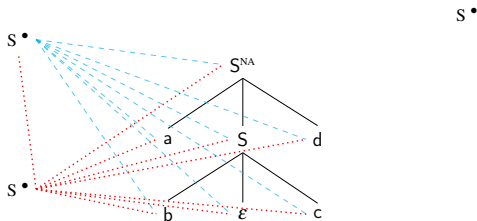
We can increase the dimensionality of a grammar



d-dimensional Local Structure

- d-dimensional mother
- $y^d$ : (d)-yield

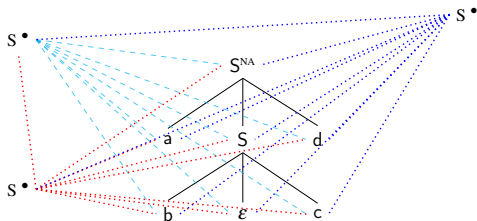
We can increase the dimensionality of a grammar



d-dimensional Local Structure

- d-dimensional mother
- $y_d^{d-1}$ : (d-1)-yield

We can increase the dimensionality of a grammar



## d-dimensional Local Structure

- d-dimensional mother
- $y d^{d-1}$ : (d-1)-yield

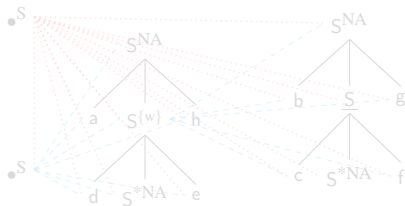


# A 4d Example

## The 8-language

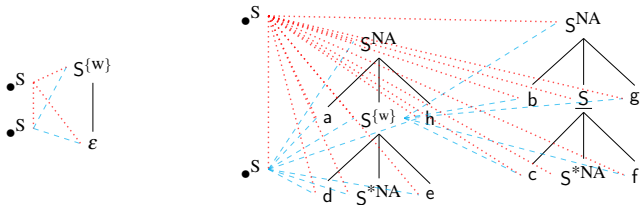
$$a^n b^n c^n d^n e^n f^n g^n h^n$$


(u)



(w)

## The 8-language

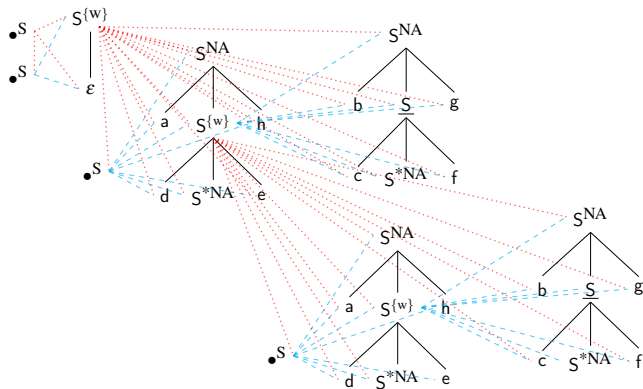
$$a^n b^n c^n d^n e^n f^n g^n h^n$$


(u)

(w)

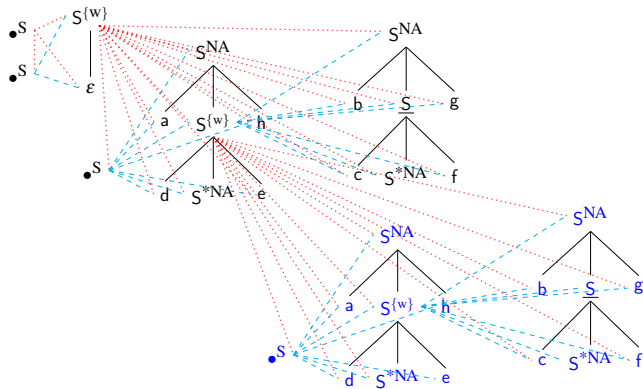
# The 8-language: a derivation

## The 4-d structure



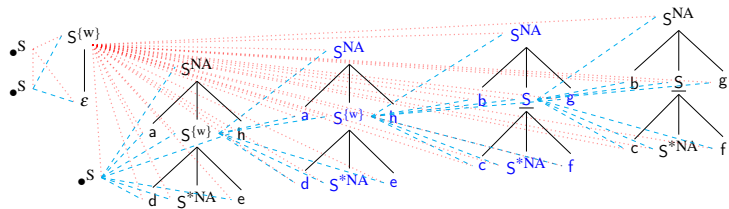
# The 8-language: a derivation

## The 4-d structure



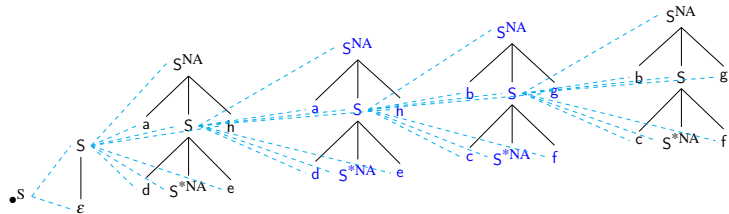
# The 8-language: a derivation

The 3-d yield ...



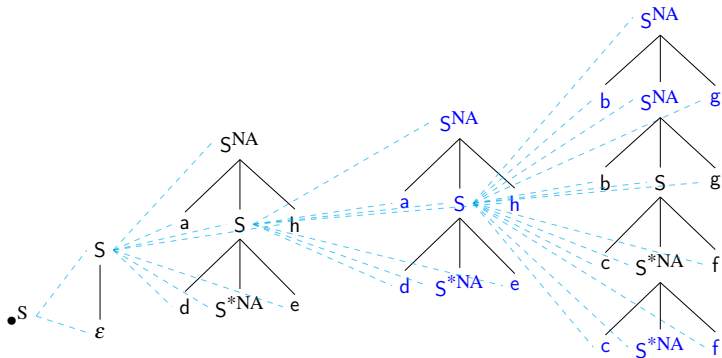
# The 8-language: a derivation

The 3-d yield!



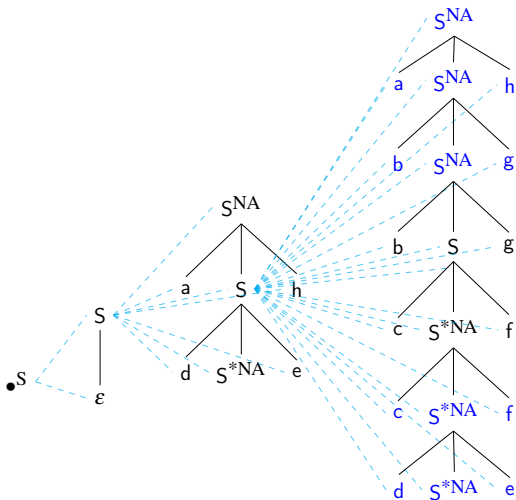
# The 8-language: a derivation

The 2-d yield ...



# The 8-language: a derivation

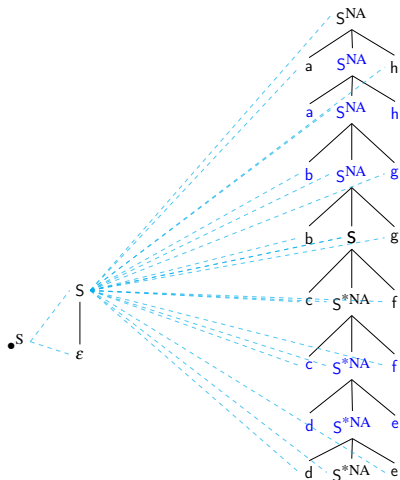
The 2-d yield ...





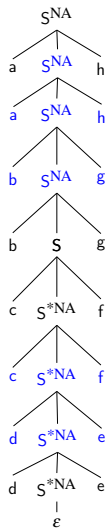
# The 8-language: a derivation

The 2-d yield ...



# The 8-language: a derivation

The 2-d yield!



## The Road so far

- Substitution as Adjunction
- TAGs as natural 3-d structures
- the generalization to higher dimensions is easy

## Next Steps

We can generalize existing proofs to multidimensional structures:

- $d$ -TAGs are  $(d+1)$ -TSGs
- $d$ -TAGs strongly lexicalize  $d$ -TSGs
- $(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## The Road so far

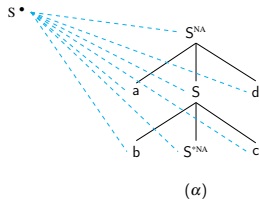
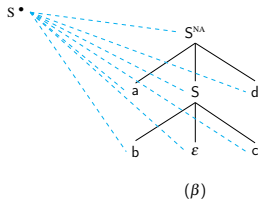
- Substitution as Adjunction
- TAGs as natural 3-d structures
- the generalization to higher dimensions is easy

## Next Steps

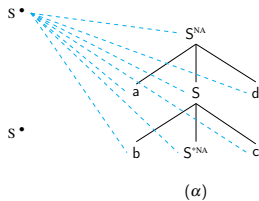
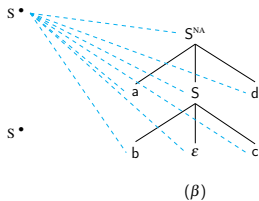
We can generalize existing proofs to multidimensional structures:

- $d$ -TAGs are  $(d+1)$ -TSGs
- $d$ -TAGs strongly lexicalize  $d$ -TSGs
- $(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

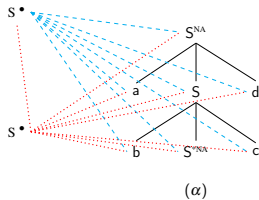
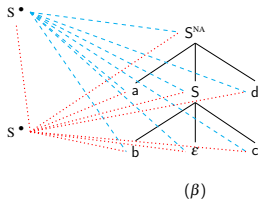
We can easily convert a 3d grammar into a 4d one



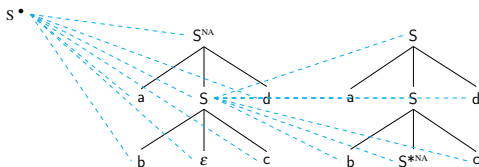
We can easily convert a 3d grammar into a 4d one



We can easily convert a 3d grammar into a 4d one



We can show that adjunction in  $d$  is substitution in  $(d+1)$

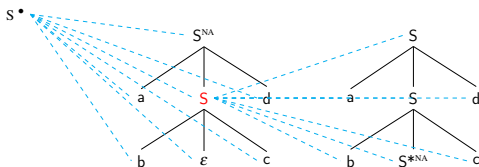


Properties of  $S$

2-dimensional mother  $\Rightarrow$  3d adjunction



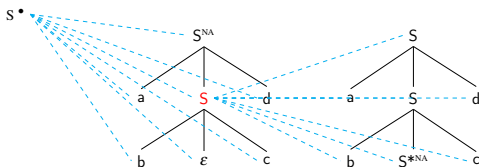
We can show that adjunction in  $d$  is substitution in  $(d+1)$



Properties of  $S$

2-dimensional mother  $\Rightarrow$  3d adjunction

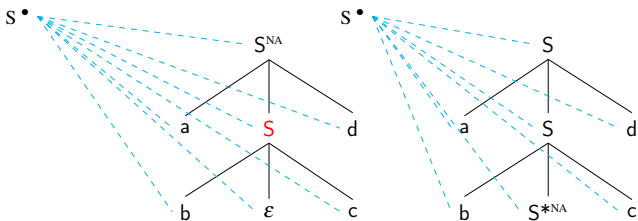
We can show that adjunction in  $d$  is substitution in  $(d+1)$



## Properties of $S$

2-dimensional mother  $\Rightarrow$  **3d adjunction**

# $d$ -TAGs are $(d+1)$ -TSGs



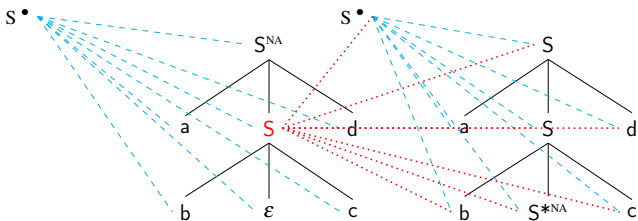
## Properties of $S$

3-dimensional leaf  $\Rightarrow$   $4d$  substitution

## Reminder: Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite nodes that are not mothers in the  $(d-1)$ -dimension

# $d$ -TAGs are $(d+1)$ -TSGs



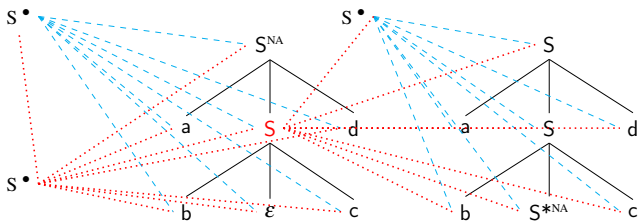
## Properties of $S$

3-dimensional leaf  $\Rightarrow$   $4d$  substitution

## Reminder: Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite nodes that are not mothers in the  $(d-1)$ -dimension

# $d$ -TAGs are $(d+1)$ -TSGs



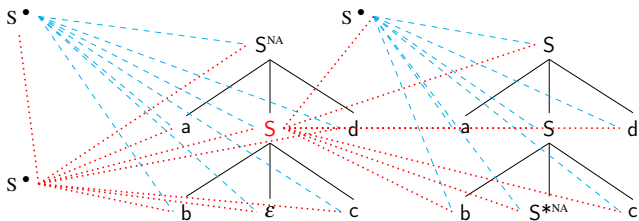
## Properties of $S$

3-dimensional leaf  $\Rightarrow$   $4d$  substitution

## Reminder: Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite nodes that are not mothers in the  $(d-1)$ -dimension

# $d$ -TAGs are $(d+1)$ -TSGs



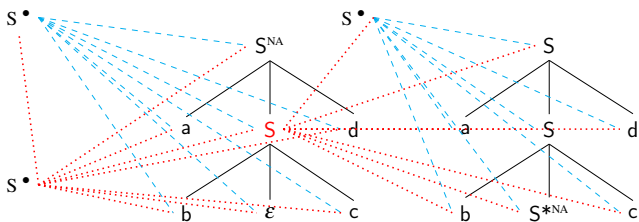
## Properties of $S$

3-dimensional leaf  $\Rightarrow$  **4d substitution**

Reminder: Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite nodes that are not mothers in the  $(d-1)$ -dimension

# $d$ -TAGs are $(d+1)$ -TSGs



## Properties of $S$

3-dimensional leaf  $\Rightarrow$  **4d substitution**

## Reminder: Tree Substitution Grammar (TSG)

A restricted TAG where all licit instances of adjunction only rewrite nodes that are not mothers in the  $(d-1)$ -dimension

[Schabes, 1990]

TAGs strongly lexicalize TSGs.

## A Lexicalization Procedure

Consider a TSG  $G$ :

- 1 Divide  $G$  in recursive and non-recursive;
- 2 Construct the set  $I_{lex}$  of initial trees;
- 3 Construct the set  $A$  of auxiliary trees.

We can extend the procedure to  $d$ -dimensional grammars



[Schabes, 1990]

TAGs strongly lexicalize TSGs.

## A Lexicalization Procedure

Consider a TSG  $G$ :

- 1 Divide  $G$  in recursive and non-recursive;
- 2 Construct the set  $I_{lex}$  of initial trees;
- 3 Construct the set  $A$  of auxiliary trees.

We can extend the procedure to  $d$ -dimensional grammars

[Schabes, 1990]

TAGs strongly lexicalize TSGs.

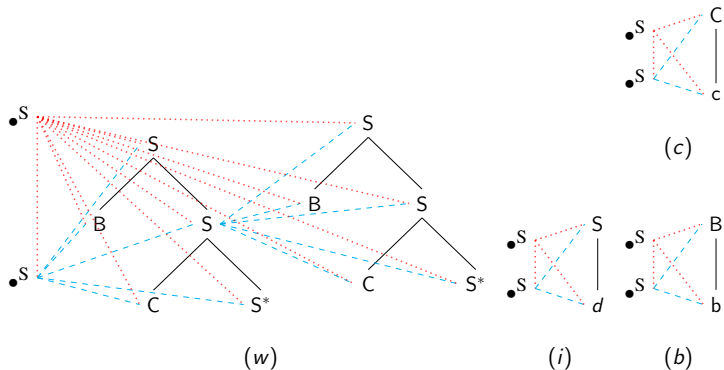
## A Lexicalization Procedure

Consider a TSG  $G$ :

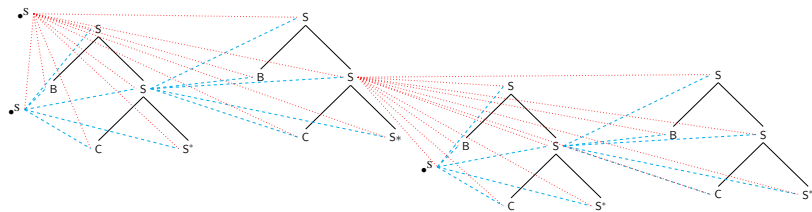
- 1 Divide  $G$  in recursive and non-recursive;
- 2 Construct the set  $I_{lex}$  of initial trees;
- 3 Construct the set  $A$  of auxiliary trees.

We can extend the procedure to  $d$ -dimensional grammars

# A non lexicalized 4d-TSG

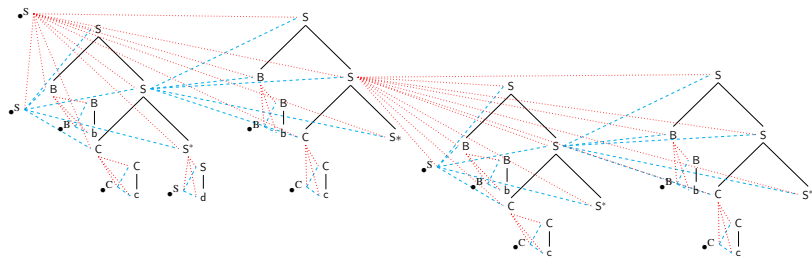


## Substitution in 4d



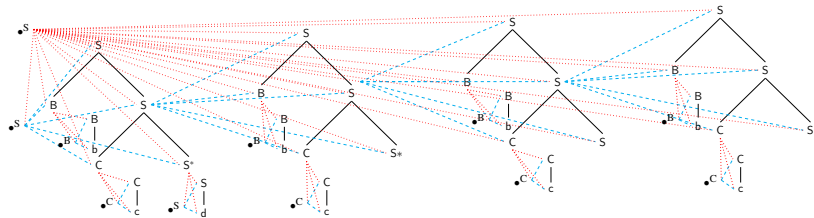
# A non lexicalized 4d-TSG: a derivation

## Substitution in 4d



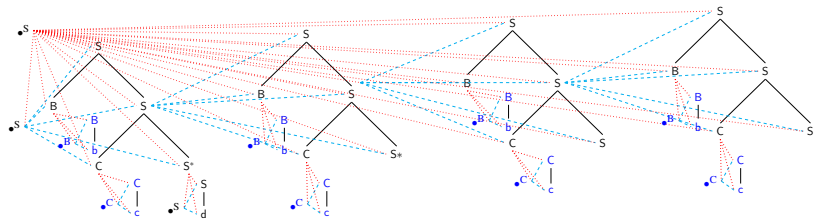
# A non lexicalized 4d-TSG: a derivation

The 3d yield...



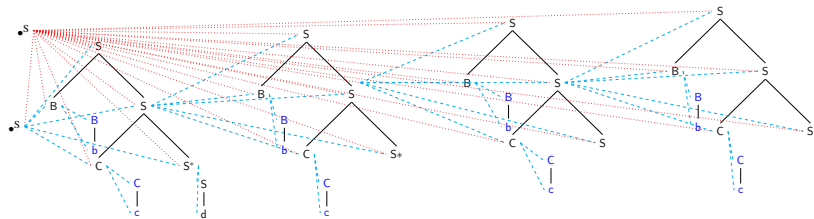
# A non lexicalized 4d-TSG: a derivation

The 3d yield...



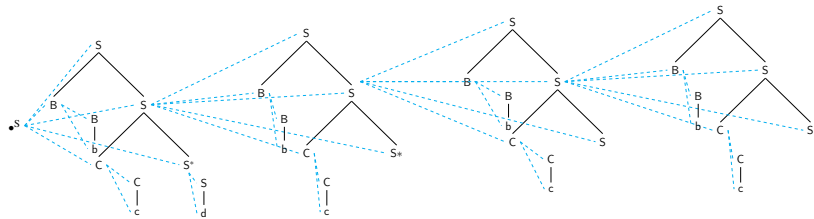
# A non lexicalized 4d-TSG: a derivation

The 3d yield...



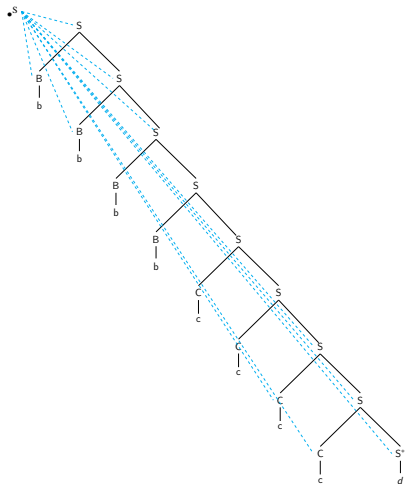


The 3d yield!



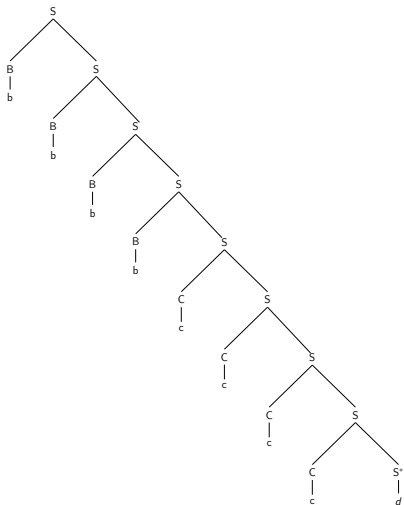
# A non lexicalized 4d-TSG: a derivation

The 2d yield ...

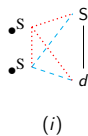
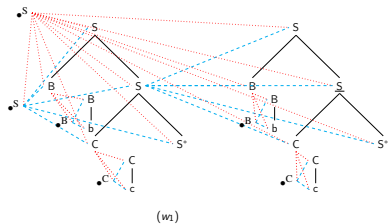
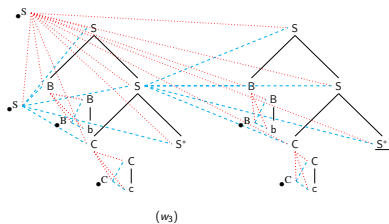
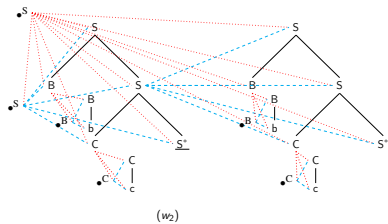


# A non lexicalized 4d-TSG: a derivation

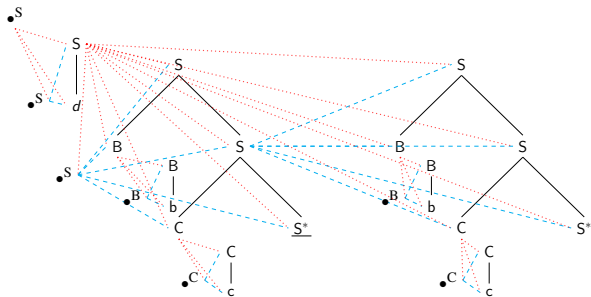
The 2d yield!



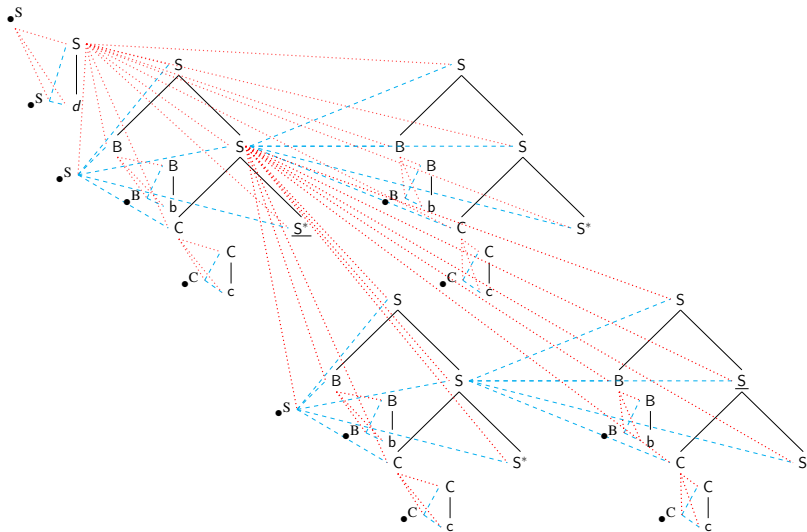
# The lexicalized 4d-TAG



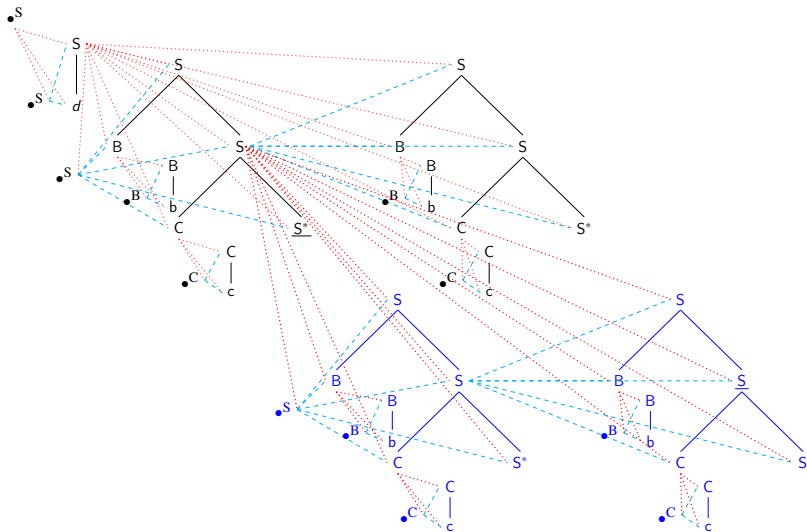
# The lexicalized 4d-TAG: a derivation



# The lexicalized 4d-TAG: a derivation

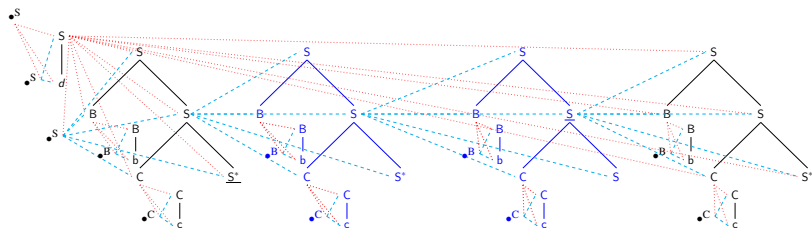


# The lexicalized 4d-TAG: a derivation



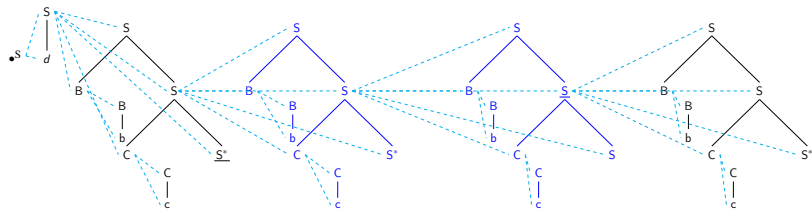
# The lexicalized 4d-TAG: a derivation

The 3d yield ...





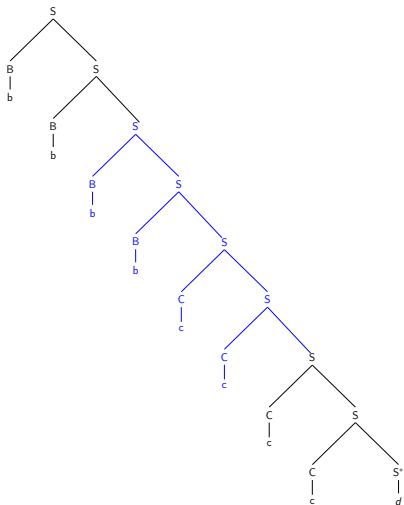
## The 3d yield!





# The lexicalized 4d-TAG: a derivation

The 2d yield!



## Proposition

For each finitely ambiguous  **$d$ -dimensional** TSG that does not generate the empty string and contains only useful trees, there is a strongly equivalent  **$d$ -dimensional** Lexicalized TAG.

but

$d$ -TSGs are equivalent to  $(d - 1)$ -TAGs.

## Proposition

$(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## Proposition

For each finitely ambiguous  **$d$ -dimensional** TSG that does not generate the empty string and contains only useful trees, there is a strongly equivalent  **$d$ -dimensional** Lexicalized TAG.

but

$d$ -TSGs are equivalent to  $(d - 1)$ -TAGs.

## Proposition

$(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## Proposition

For each finitely ambiguous  **$d$ -dimensional** TSG that does not generate the empty string and contains only useful trees, there is a strongly equivalent  **$d$ -dimensional** Lexicalized TAG.

but

$d$ -TSGs are equivalent to  $(d - 1)$ -TAGs.

## Proposition

**$(d+1)$** -TAGs strongly lexicalize  **$d$** -TAGs

- TAGs can be generalized to higher dimensional trees [Rogers, 2003]
- TAGs strongly lexicalize CFGs/TSGs [Schabes, 1990]

⇒  $(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## TAGs as higher dimensional-trees

- lifting of existing results is straightforward
- increase in generative power
- what about parsing?

- TAGs can be generalized to higher dimensional trees [Rogers, 2003]
- TAGs strongly lexicalize CFGs/TSGs [Schabes, 1990]

⇒  $(d+1)$ -TAGs strongly lexicalize  $d$ -TAGs

## TAGs as higher dimensional-trees

- lifting of existing results is straightforward
- increase in generative power
- what about parsing?





Fujiyoshi, A. (2004).

Epsilon-free grammars and lexicalized grammars that generate the class of the mildly contextsensitive languages.

*In Proceedings of the 7th International Workshop on Tree Adjoining Grammar and Related Formalisms*, pages 16–23.



Kuhlmann, M. and Satta, G. (2012).

Tree-adjoining grammars are not closed under strong lexicalization.

*Computational Linguistics*, 38:617–629.



Maletti, A. and Engelfriet, J. (2012).

Strong lexicalization of tree adjoining grammars.

*In Proceedings of the 50th Annual Meeting of the Association for Computational Linguistics: Long Papers - Volume 1, ACL '12*, pages 506–515.



Rogers, J. (1998).

On defining TALs with logical constraints.

In Abeillé, A., Becker, T., Rambow, O., Satta, G., and Vijay-Shanker, K., editors, *Fourth International Workshop on Tree Adjoining Grammars and Related Frameworks (TAG+4)*, pages 151–154.



Rogers, J. (2003).

Syntactic structures as multi-dimensional trees.

*Research on Language and Computation*, 1:265–305.



Schabes, Y. (1990).

*Mathematical and Computational Aspects of Lexicalized Grammars*.

PhD thesis, Philadelphia, PA, USA.



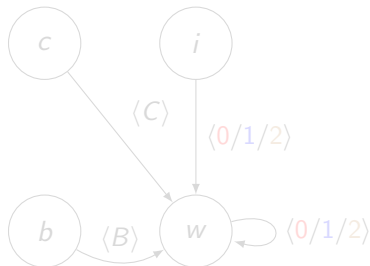
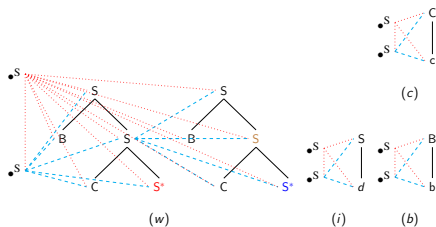
Schabes, Y., Abeillé, A., and Joshi, A. K. (1988).

Parsing strategies with 'lexicalized' grammars: Application to tree adjoining grammars.

Technical Report MS-CIS-88-65, Department of Computer & Information Science, University of Pennsylvania, Philadelphia, PA.

# Lexicalization of a $d$ -TSG: Step 1

## Step 1: Determine Recursion



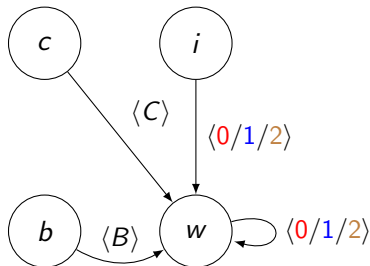
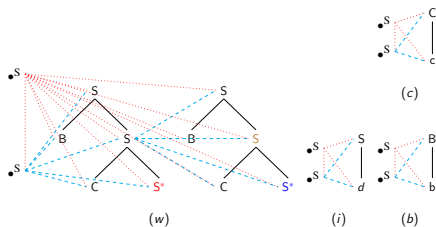
The TSG is Partitioned in Two Sets

$$NR = \{b, c, i\}$$

$$R = \{I - NR\} = \{w\}$$

# Lexicalization of a $d$ -TSG: Step 1

## Step 1: Determine Recursion



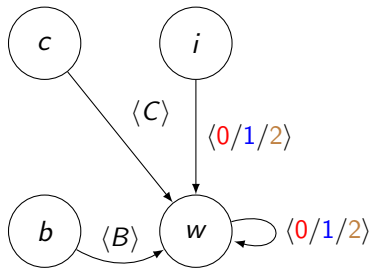
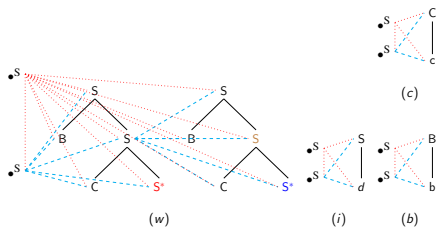
The TSG is Partitioned in Two Sets

$$NR = \{b, c, i\}$$

$$R = \{I - NR\} = \{w\}$$

# Lexicalization of a $d$ -TSG: Step 1

## Step 1: Determine Recursion



## The TSG is Partitioned in Two Sets

$$NR = \{b, c, i\}$$

$$R = \{I - NR\} = \{w\}$$

Step 2: Determine the set  $I_{lex}$ .

- $T(NR)$  : the closure of  $NR$  under adjunction

$I_{lex}$

is the maximal subset of  $T(NR)$  that only contains  $d$ -trees whose root is labeled by the start category  $S$ .

$$I_{lex} = \{i\}$$

Step 2: Determine the set  $I_{lex}$ .

- $T(NR)$  : the closure of  $NR$  under adjunction

$I_{lex}$

is the maximal subset of  $T(NR)$  that only contains  $d$ -trees whose root is labeled by the start category  $S$ .

$$I_{lex} = \{i\}$$



Step 2: Determine the set  $I_{lex}$ .

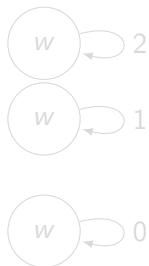
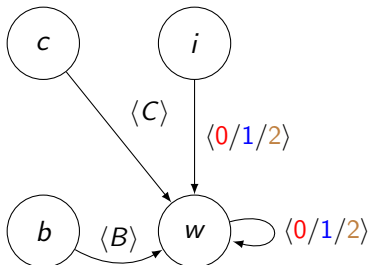
- $T(NR)$  : the closure of  $NR$  under adjunction

$I_{lex}$

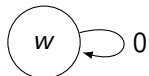
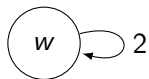
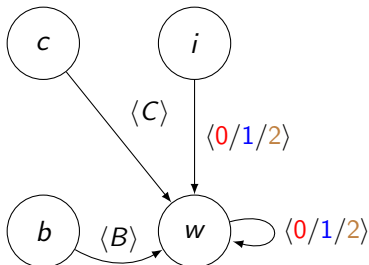
is the maximal subset of  $T(NR)$  that only contains  $d$ -trees whose root is labeled by the start category  $S$ .

$$I_{lex} = \{i\}$$

## Step 3: Compute Base Cycles



## Step 3: Compute Base Cycles

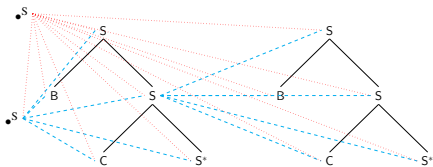
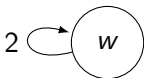
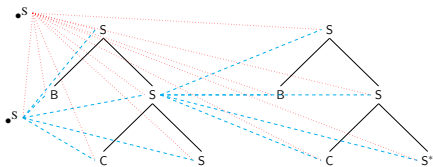
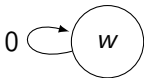


# Lexicalization of a $d$ -TSG: Step 4

## Step 4:

### Determine $A_{lex}$

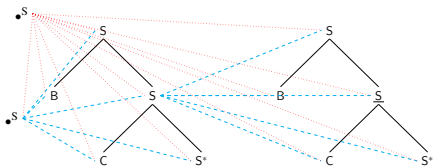
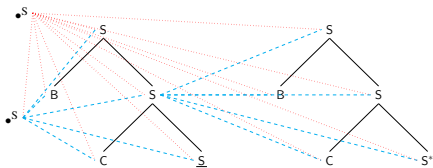
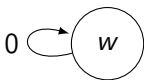
- expand base cycles;
- relabel  $3d$  foot node;
- exhaustive substitution;



# Lexicalization of a $d$ -TSG: Step 4

## Step 4: Determine $A_{lex}$

- expand base cycles;
- relabel  $3d$  foot node;
- exhaustive substitution;

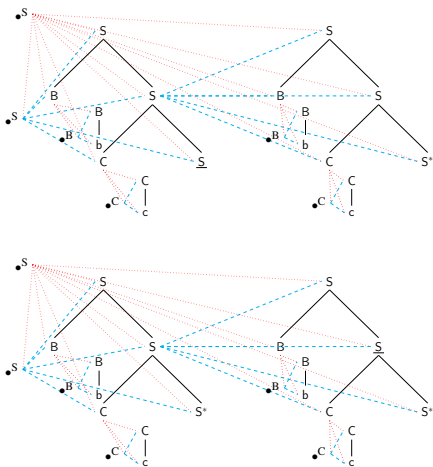
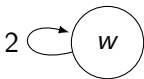
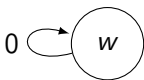


# Lexicalization of a $d$ -TSG: Step 4

## Step 4:

### Determine $A_{lex}$

- expand base cycles;
- relabel  $3d$  foot node;
- exhaustive substitution;

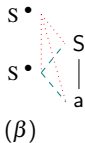
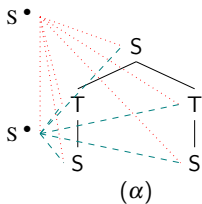


# One final question

Are  $d$ -dimensional TAGs closed under strong lexicalization?

[Kuhlmann and Satta, 2012]

TAGs are not closed under strong lexicalization



$$\begin{aligned} (\alpha) \quad S^{NA} - \left( \begin{array}{c} \bullet S \\ S^{NA} \\ S^{OA} \\ T^{NA} \\ S^{OA} \\ T^{NA} \end{array} \right) - S^{NA} \\ (\beta) \quad S^{NA} - \left( \begin{array}{c} \bullet S \\ S^{NA} \\ a \end{array} \right) - S^{NA} \\ (\gamma) \quad S^{OA} - \varepsilon \end{aligned}$$

*Excess*

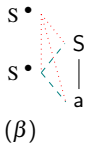
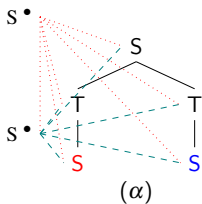
measures the distance between a root node and a terminal node

# One final question

Are  $d$ -dimensional TAGs closed under strong lexicalization?

[Kuhlmann and Satta, 2012]

TAGs are not closed under strong lexicalization



$$\begin{aligned} (\alpha) \quad S^{\text{NA}} - \left( \begin{array}{c} \bullet S \\ S^{\text{NA}} \\ S^{\text{OA}} \\ T^{\text{NA}} \\ S^{\text{OA}} \\ T^{\text{NA}} \end{array} \right) - S^{\text{NA}} \\ (\beta) \quad S^{\text{NA}} - \left( \begin{array}{c} \bullet S \\ S^{\text{NA}} \\ a \end{array} \right) - S^{\text{NA}} \\ (\gamma) \quad S^{\text{OA}} - \varepsilon \end{aligned}$$

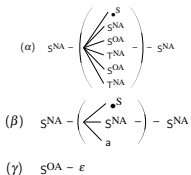
*Excess*

measures the distance between a root node and a terminal node



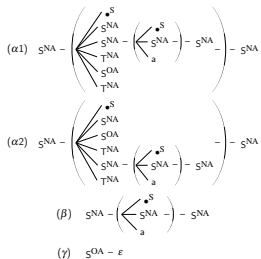
# $d$ -TAGs are not closed under strong lexicalization

## Non Lexicalized



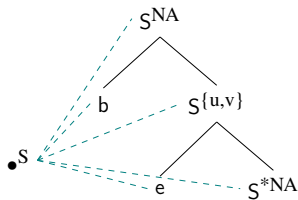
*max. excess* of node  $a$  is  
**unbounded**

## Lexicalized

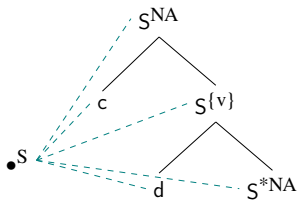


*max excess* of node  $a$  is  
**2**

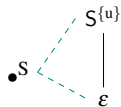
# Increasing the power by increasing the dimensionality: Ex. 2



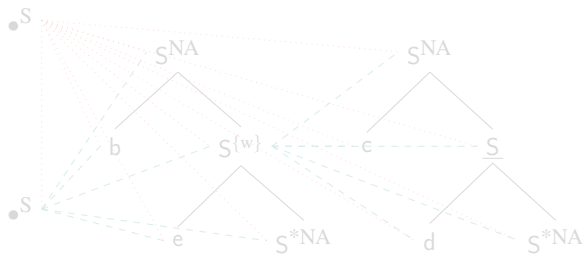
$u$



$v$



$i$

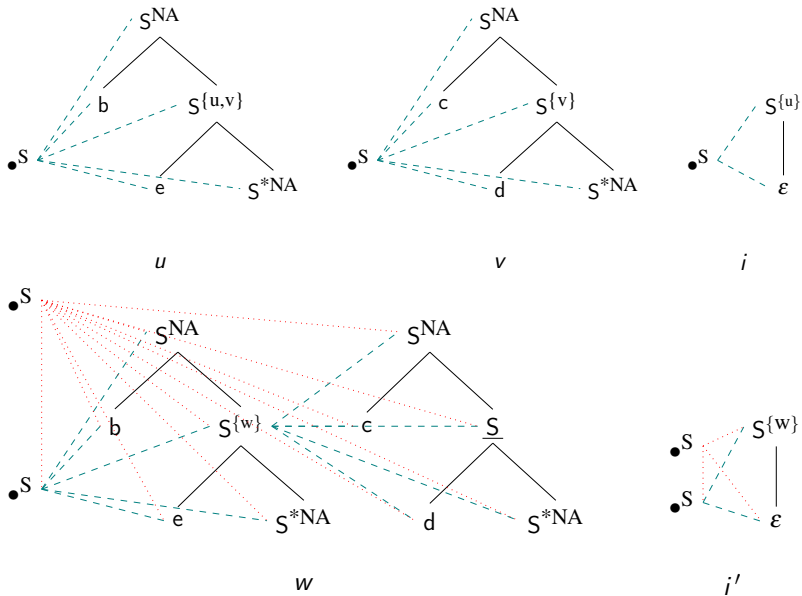


$w$



$i'$

# Increasing the power by increasing the dimensionality: Ex. 2



# Map of Existing Results

