# Closure Properties of Minimalist Derivation Tree Languages 

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## Outline

(1) Regular Tree Languages
(2) Minimalist Grammars

- Derived Tree Languages
- Derivation Trees
(3) Closure Properties of Minimalist Derivation Tree Languages
- Non-Closure Under Intersection with REG
- P-Closure Under Intersection with REG
- Further P-Closure Properties
(4) Applications


## Regular $=$ Recognized by Bottom-Up Tree Automaton

- Bottom-up tree automata generalize finite-state automata from strings to trees.
- Only significant change in the transition function: domain extended from pairs of symbols and states to $n+1$ tuples $\left\langle q_{1}, \ldots, q_{n}, \sigma^{(n)}\right\rangle$, where $\sigma^{(n)}$ is a symbol of arity $n \geq 0$.


## Deterministic Bottom-Up Tree Automata

A deterministic bottom-up tree automaton is a 4-tuple $A:=\langle\Sigma, Q, F, \delta\rangle$, where

- $\Sigma$ is a ranked alphabet,
- $Q$ is a finite set of states (i.e. of unary symbols $q \notin \Sigma$ ),
- $F \subseteq Q$ is the set of final states,
- $\delta:\left(\bigcup_{n \geq 0} Q^{n} \times \Sigma^{(n)}\right) \rightarrow Q$ is the transition function.


## ODD: A Regular Tree Language

Let $O D D$ be the language of all (at most) binary branching trees over alphabet $\Sigma:=\left\{a^{(0)}, a^{(1)}, a^{(2)}\right\}$ such that every tree has an odd number of nodes.

## Automaton for ODD

$A_{O D D}:=\left\langle\left\{a^{(0)}, a^{(1)}, a^{(2)}\right\},\{O, E\},\{O\}, \delta\right\rangle$, where $\delta$ is given by the following rules:

$$
\begin{aligned}
a \rightarrow O & (O, O, a) \rightarrow O \\
(O, a) \rightarrow E & (O, E, a) \rightarrow E \\
(E, a) \rightarrow O & (E, O, a) \rightarrow E \\
& (E, E, a) \rightarrow O
\end{aligned}
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Two Runs of $A_{O D D}$


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## Regular Tree Languages/Automata for Linguistics

- Just like regular string languages, regular tree languages are very well-behaved mathematically
$\Rightarrow$ attractive from a computational perspective
- Almost all parts of Government-and-Binding theory can be expressed by bottom-up automata (Rogers 1998) $\Rightarrow$ regular tree languages sufficiently powerful for most syntactic generalizations/constraints
- But the string yield of a regular tree language is context-free $\Rightarrow$ too weak for natural language
- Minimalist grammars (MGs) generate MCFLs, yet can be fully specified by regular tree languages. But is it possible to add regular constraints to MGs without increasing their weak generative capacity?


## The Atoms of a Minimalist Grammar

## Minimalist Grammars (MGs; Stabler 1997)

An MG is a 5 -tuple $G:=\langle\Sigma$, Feat, $F$, Lex, Op $\rangle$, where

- $\Sigma$ is an alphabet,
- Feat is a non-empty finite set of
- category features $f$,
- selector features $=f$,
- movement licensee features $-f$,
- movement licensor features $+f$,
- $F \subseteq$ Feat is a set of final category features,
- the lexicon Lex is a finite subset of $\Sigma^{*} \times$ Feat $^{+}$,
- Op $:=\{$ merge, move $\}$ is the set of structure-building operations.

For every MGs it suffices to specify Lex and $F$.

## Bare Phrase Structure Trees

- My definition of merge and move is tree-based.
- It builds on the notion of Bare Phrase Structure trees and Headedness.


## Extended Lexicon

Given a lexicon Lex, its extended lexicon Elex is the smallest set such that, for $\sigma \in \Sigma^{*}, f \in$ Feat, and $\delta \in$ Feat*

- $I \in L e x \rightarrow I \in$ Elex
- I $:=\langle\sigma, f \delta\rangle \in$ Elex $\rightarrow I^{\prime}:=\langle\sigma, \delta\rangle \in$ Elex


## Bare Phrase Structure Trees (BPS Trees)

The set of BPS trees over Elex consists of all strictly binary branching trees over the ranked alphabet $\left\{\left\langle^{(2)},>{ }^{(2)}\right\} \cup\left\{I^{(0)} \mid I \in\right.\right.$ Elex $\}$.

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## Headedness

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Given a BPS tree $t$, the head of $t$ is given by

$$
\operatorname{head}(t):= \begin{cases}t & \text { if } t \in \text { Elex } \\ \operatorname{head}\left(t_{1}\right) & \text { if } t:=>t_{1}^{\prime} / t_{2} \\ \operatorname{head}\left(t_{2}\right) & \text { if } t:=t_{t_{1}}^{>} t_{2}\end{cases}
$$

Notation $t^{\delta}$ denotes that head $(t)$ carries feature string $\delta$

## Defining Merge \& Move

Let $\gamma, \delta \in$ Feat ${ }^{*}$.

$$
\operatorname{merge}\left(s^{=f \gamma}, t^{f \delta}\right):= \begin{cases}s^{\gamma}>t^{\delta} & \text { if } s \in \text { Elex } \\ t^{\delta^{\delta} / \backslash} s^{\gamma} & \text { otherwise }\end{cases}
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## Shortest Move Constraint (SMC)

Every tree $s^{+f \gamma}$ in the domain of move has exactly one subtree $t$ such that the first feature of head $(t)$ is $-f$.

Thanks to the SMC, both Merge and Move are deterministic.

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## Derived Tree Language \& Expressivity

## Derived Tree Language

The tree language $L(G)$ derived by MG $G$ with lexicon $\operatorname{Lex}_{G}$ is the largest set of BPS trees such that

- $L(G) \subseteq \operatorname{closure}\left(L^{\prime}{ }_{G},\{\right.$ merge, move $\left.\}\right)$,
- for every $t \in L(G)$, there is some $f \in F_{G}$ such that the feature component of head $(t)$ consists only of $f$,
- all other leaves have an empty feature component.


## Generated String Language <br> The string language generated by MG $G$ is the string yield of $L(G)$.

## Theorem (Harkema 2001; Michaelis 1998, 2001)

MCFGs and MGs are weakly equivalent.

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## A Toy Example (Without Recursion)

## MG with $F=\{C\}$

```
men :: N
the :: = N D
what :: D - wh
```

like $::=\mathrm{D}=\mathrm{D}$ V
$\varepsilon::=\mathrm{V}$ C
do :: $=\mathrm{V}+\mathrm{wh} \mathrm{C}$

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\begin{array}{ll}
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## Derivation Trees

## Useful Fact

Every MG is fully specified by its set of derivation trees, which is regular (Kobele et al. 2007).


## Defining Derivation Trees: The Intuition

- Defining well-formed derivation trees of MG G only requires keeping track of the feature calculus $\Rightarrow$ deterministic bottom-up automaton with sequences of feature strings as states (and $F_{A}:=\left\{\langle f\rangle \mid f \in F_{G}\right\}$ )
- Due to the SMC, the number of feature strings per state is bounded $\Rightarrow$ finite number of states



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## Non-Closure Under Intersection with REG

## Theorem

The class of MDTLs is not closed under intersection with regular tree languages.

## Proof.

- Let ODD contain all trees with an odd number of nodes.
- Let $G$ be the $M G$ given by $F_{G}=\{c\}$ and $L e x_{G}$ :

a : : a $-k$
- Then there are derivation trees $s$ and $t$ in the closure of $\operatorname{Lex}_{G}$ under \{merge, move $\}$ that both end in a final category and contain the same lexical items. It is easy to see that $s \in \operatorname{mder}\left(G^{\prime}\right)$ iff $t \in \operatorname{mder}\left(G^{\prime}\right)$ for any MG $G^{\prime}$, yet $s \notin \operatorname{mder}(G) \cap O D D \ni t$.


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\mathrm{a}:: \mathrm{a} \quad \mathrm{~b}::=\mathrm{a}=\mathrm{a}+\mathrm{k} \mathrm{a} \quad \mathrm{c}::=\mathrm{a} \mathrm{c}
$$

a :: a - k

- Then there are derivation trees $s$ and $t$ in the closure of Lex $_{G}$ under $\{$ merge, move $\}$ that both end in a final category and contain the same lexical items. It is easy to see that $s \in \operatorname{mder}\left(G^{\prime}\right)$ iff $t \in m d e r\left(G^{\prime}\right)$ for any MG $G^{\prime}$, yet $s \notin \operatorname{mder}(G) \cap O D D \ni t$.


## Choice of $s$ and $t$



$$
\begin{gathered}
s=u \notin O D D \\
t=u+v \in O D D
\end{gathered}
$$

## Defining P-Closure

## Projection

Let $\lambda: \Sigma \rightarrow \Omega$ be a many-to-one map between alphabets, and $\pi$ its extension from alphabets to trees.
Tree $t$ is a projection of $s$ iff there is a $\pi$ such that $t=\pi(s)$. The notion extends to tree languages in the natural way.

## P[rojection]-Closure

Given a class of languages $\mathcal{L}$ and an operation $O$, $\mathcal{L}$ is $p$-closed under $O$ iff the result of applying $O$ to some $L \in \mathcal{L}$ is a projection of some $L^{\prime} \in \mathcal{L}$.

## P-Closure Under Intersection with REG

## Theorem (REG Intersection P-Closure)

The class of MDTLs over alphabet $\Sigma$ and features Feat is p-closed under intersection with regular tree languages.

## Outline of Proof

- Inspired by Thatcher's theorem (translate recognizable sets into local ones by incorporating states into alphabet)
- Crux: Internal node labels of a derivation tree cannot be refined $\Rightarrow$ slices as a way of relating interior nodes to features on lexical items
- Procedure for refining category and selector features so that they incorporate states of the deterministic bottom-up automaton recognizing regular language


## Slices

Intuitively, slices are the derivation tree equivalent of phrasal projection: Each slice marks the subpart of the derivation that a lexical item has control over by virtue of its selector and licensor features.

## Slices

Given a derivation tree $t$ and lexical item / occurring in $t$, slice $(I)$ is defined as follows:

- $I \in \operatorname{slice}(I)$,
- if node $n$ of $t$ immediately dominates a node $s \in$ slice $(I)$, then $n \in \operatorname{slice}(I)$ iff the operation denoted by the label of $n$ erased a selector or licensor feature of $I$.

The unique $n \in$ slice $(I)$ that isn't (properly) dominated by any $n^{\prime} \in$ slice $(I)$ is called the slice root of $I$.

## Example of Slices



## Simple Facts About Slices

- Every node of a derivation tree belongs to some slice.
- Slices are continuous.
- Moving from slice( $I$ ) to slice $\left(I^{\prime}\right)$ such that $I^{\prime}$ was selected by $I$, one eventually reaches a slice of size 1 .


## Category Refinement Strategy

- Assume we are given an MG G and deterministic bottom-up automaton $A$.
- Subscript interior node labels with state of automaton, following Thatcher's strategy.
- Move subscript from slice root of lexical item to its category feature.
- Refine selection features accordingly.
- The set of final categories of the new MG $G^{\prime}$ is $\left\{c_{q} \mid c \in F_{G}, q \in F_{A}\right\}$.
- Note that only finitely many combinations of slices and states need to be considered, so the procedure can be carried out efficiently.


## Two Examples of Category Refinement



## MG $G^{\prime}$ for $G \cap O D D$

$$
\begin{array}{ll}
a: a_{0} & b: \because=a_{0}=a_{0}+k a_{e} \\
a \because a_{0}-k & b:=a_{0}=a_{e}+k a_{0} \\
b: \because=a_{e}=a_{0}+k a_{0} \\
& b: \because=a_{e}=a_{e}+k a_{e}
\end{array}
$$

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## MG $G^{\prime}$ for $G \cap O D D$

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\begin{array}{ll}
a: \therefore a_{0} & b::=a_{0}=a_{0}+k a_{e} \\
a \because a_{0}-k & b: \because=a_{0}=a_{e}+k a_{0} \\
b: \because=a_{e}=a_{0}+k a_{0} \\
b: c_{0} \\
b=a_{e}=a_{e}+k a_{e}
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$$
\begin{aligned}
& \text { a }:: a_{o} \\
& \mathrm{~b}::=\mathrm{a}_{\mathrm{o}}=\mathrm{a}_{\mathrm{o}}+\mathrm{k} \mathrm{a}_{\mathrm{e}} \\
& \mathrm{c}::=\mathrm{a}_{\mathrm{o}} \mathrm{c}_{\mathrm{o}} \\
& \text { a :: } a_{o}-k \\
& \mathrm{~b}::=\mathrm{a}_{\mathrm{o}}=\mathrm{a}_{\mathrm{e}}+\mathrm{k} \mathrm{a}_{\mathrm{o}} \\
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## Correctness of Procedure

- Suppose that $m \operatorname{der}\left(G^{\prime}\right) \neq \operatorname{mder}(G) \cap L(A)$.
- Then there must be some tree $t$ such that $t \in \operatorname{mder}\left(G^{\prime}\right)$ iff $\pi(t) \notin m \operatorname{der}(G) \cap L(A)$. So head $(t)$ has a category feature $c_{q}$, but $A$ does not assign state $q$ to the root of $\pi(t)$.
- Since $A$ is deterministic, such a situation may arise only if $A$ entered the slice in a state that differs from the subscripts on the corresponding selector feature of head $(t)$.
- By induction on slices, we eventually reach a slice of size 1 to which $A$ assigned a state that differs from the subscript of the category feature of its lexical item. But $A$ is deterministic. Contradiction.


## Further P-Closure Properties

## P-Closure Corollaries

- The class of MDTLs over $\Sigma$, Feat is p-closed under
- intersection,
- relative complement.
- Given lexicon Lex, the class of MDTLs over subsets of Lex is p -closed under
- complement,
- union.
- For every regular tree language $L$ and linear transduction $\tau$ with an MDTL as its co-domain, it holds that $\tau(L)$ is a projection of some MDTL.


## Minimalist Grammars with Regular Control

## Minimalist Grammars with Regular Control (MGRCs)

An MG is a 6 -tuple $G:=\langle\Sigma$, Feat, $F$, Lex, $O p, \mathcal{R}\rangle$, where

- $\Sigma$, Feat, F, Lex, and $O p$ are defined as usual, and
- $\mathcal{R}$ is a finite collection of regular tree languages.

Its controlled derivation tree language is $\operatorname{cder}(G):=m \operatorname{der}(G) \cap \mathcal{R}$. The derived tree language of $G$ (and its string yield) are obtained from $\operatorname{cder}(G)$ via the mbutt of Kobele et al. (2007).

- MGRCs are more succinct than their refined equivalent.
- Given a lexicon Lex and $n \geq 0$, let Lex ${ }^{(n)}:=\{I \in \operatorname{Lex} \mid I$ has exactly $n$ selector features $\}$. In the worst case

$$
\mid \text { Lex }_{G^{\prime}} \mid=\sum_{i \geq 0}\left(\left|\operatorname{Lex}_{G}^{(i)}\right| \cdot|Q|^{i+1}\right)
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\left|\operatorname{Lex}_{G^{\prime}}\right|=\sum_{i \geq 0}\left(\left|\operatorname{Lex}_{G}^{(i)}\right| \cdot|Q|^{i+1}\right)
$$

## Application 1: Reference-Set Computation

- Reference-set constraints are economy conditions similar to OT: Given some input tree $t$
- compute the set of competing output candidates,
- rank them according to some economy metric,
- discard all sub-optimal candidates.
- Graf (2010a,b): Most reference-set constraints in the syntactic literature can be modelled by linear tree transductions. In particular, those constraints act as filters, so the transductions have MDTLs as their domain and co-domain.
- From the previous corollary for linear transductions it follows that the expressivity of MGs is not increased.


## Application 2: Non-Local Dependencies without Movement

- Expletive constructions in English show subject-verb agreement even though no movement seems to be involved.
(1) a. There seems to be a man in the garden.
b. There seem to be three men in the garden.
- The subject position is filled by the expletive $\Rightarrow$ arguably no movement


## Proposal

Regular constraint operating on "pseudo-features" (not part of the MG itself) $\Rightarrow$ enforce non-local dependencies without movement

- Quite generally, this allows us to enrich MGs with AGREE (Chomsky 2000).


## Application 3: Relativized Minimality

- In Minimalist syntax, the contrast below is explained by Relativized Minimality: If a movement licensor feature can be checked by two different phrases, the closer one moves.
(2) Who/what bought $t$ who/what?
(3) $\quad$ Who/What bought who/what $t$ ?
- Relativized Minimality relies on both who and what carrying a -wh feature. This idea conflicts with the SMC, and in order to derive (2), we must allow who/ what to appear without a - wh-feature. But then nothing in the MG blocks (3).


## Proposal

Moving phrase XP with feature $-f$ to ZP is banned if there is a closer YP with pseudo feature $-f$.

## Further Applications

- Island constraints
- Phases
- that-trace filter
- L-marking
- Limited feature percolation/Pied-Piping
- Control/Binding(?)


## Conclusion

- MDTLs are not closed under intersection with regular tree languages.
- However, they enjoy p-closure properties akin to regular languages:
- intersection,
- intersection with regular tree languages,
- union,
- (relative) complement,
- certain linear transductions.
- Hence, enriching MGs with regular control over their derivations does not increase their generative capacity.
- Numerous applications; in particular, ideas from model-theoretic syntax can be easily incorporated


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