REG	MGs	Closure Properties	Applications	Conclusion	References

Closure Properties of Minimalist Derivation Tree Languages

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REG 0000	MGs 00000000	Closure Properties	Applications	Conclusion O	References
Outline					



- 2 Minimalist Grammars
 - Derived Tree Languages
 - Derivation Trees

3 Closure Properties of Minimalist Derivation Tree Languages

- Non-Closure Under Intersection with REG
- P-Closure Under Intersection with REG
- Further P-Closure Properties

Applications



- **Bottom-up tree automata** generalize finite-state automata from strings to trees.
- Only significant change in the transition function: domain extended from pairs of symbols and states to n + 1 tuples $\langle q_1, \ldots, q_n, \sigma^{(n)} \rangle$, where $\sigma^{(n)}$ is a symbol of arity $n \ge 0$.

Deterministic Bottom-Up Tree Automata

- A *deterministic bottom-up tree automaton* is a 4-tuple
- $A := \langle \Sigma, Q, F, \delta \rangle$, where
 - Σ is a ranked alphabet,
 - Q is a finite set of states (i.e. of unary symbols $q \notin \Sigma$),
 - $F \subseteq Q$ is the set of final states,
 - $\delta : \left(\bigcup_{n \geq 0} Q^n \times \Sigma^{(n)} \right) \to Q$ is the transition function.

REG 0●00	MGs 00000000	Closure Properties	Applications	Conclusion O	References
ODD:	A Regul	ar Tree Lang	uage		

Let *ODD* be the language of all (at most) binary branching trees over alphabet $\Sigma := \{a^{(0)}, a^{(1)}, a^{(2)}\}$ such that **every tree has an odd number of nodes**.

Automaton for ODD

$$\begin{split} A_{ODD} &:= \left\langle \left\{ a^{(0)}, a^{(1)}, a^{(2)} \right\}, \left\{ O, E \right\}, \left\{ O \right\}, \delta \right\rangle, \\ \text{where } \delta \text{ is given by the following rules:} \end{split}$$

$$\begin{array}{ccc} a \rightarrow O & (O, O, a) \rightarrow O \\ (O, a) \rightarrow E & (O, E, a) \rightarrow E \\ (E, a) \rightarrow O & (E, O, a) \rightarrow E \\ & (E, E, a) \rightarrow O \end{array}$$

REG 0●00	MGs 00000000	Closure Properties	Applications	Conclusion O	References
ODD:	A Regul	ar Tree Lang	uage		

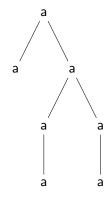
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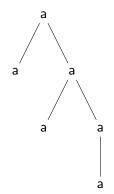
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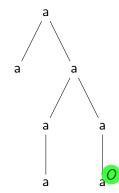
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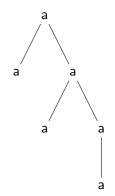
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Two R	uns of A	ODD			



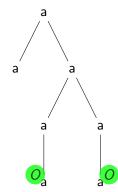


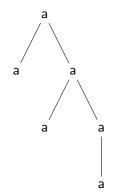
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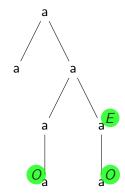


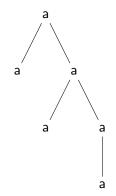
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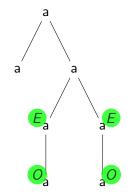


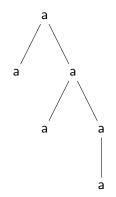
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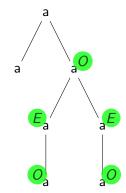


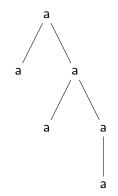
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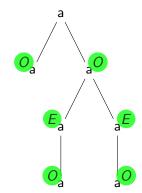


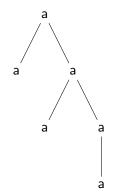
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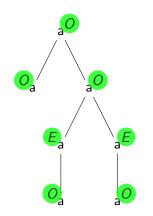


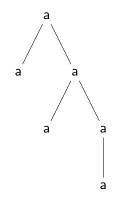
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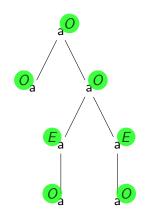


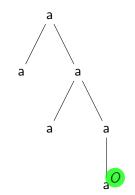
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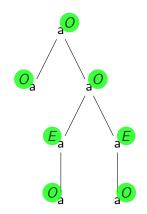


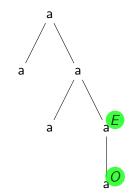
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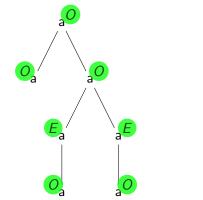


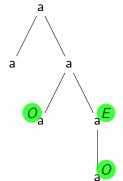
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Two F	Runs of A	ODD			



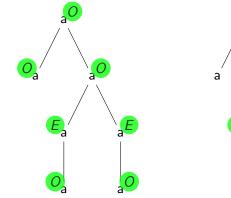


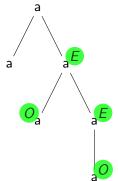
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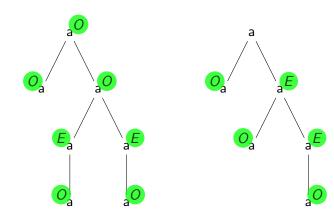


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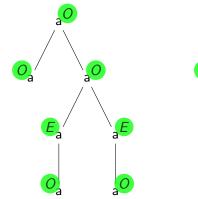


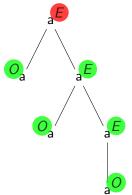


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REG 00●0	MGs 00000000	Closure Properties	Applications	Conclusion O	References
Two R	uns of A	ODD			







• Just like regular string languages, regular tree languages are very well-behaved mathematically

 \Rightarrow attractive from a computational perspective

- Almost all parts of Government-and-Binding theory can be expressed by bottom-up automata (Rogers 1998)
 ⇒ regular tree languages sufficiently powerful for most syntactic generalizations/constraints
- But the string yield of a regular tree language is context-free ⇒ too weak for natural language
- Minimalist grammars (MGs) generate MCFLs, yet can be fully specified by regular tree languages. But is it possible to add regular constraints to MGs without increasing their weak generative capacity?

The At	oms of a	Minimalist G	rammar		
REG 0000	MGs ●0000000	Closure Properties	Applications	Conclusion O	References

Minimalist Grammars (MGs; Stabler 1997)

An MG is a 5-tuple $G := \langle \Sigma, Feat, F, Lex, Op \rangle$, where

- Σ is an alphabet,
- Feat is a non-empty finite set of
 - category features f,
 - selector features = f,
 - movement licensee features -f,
 - movement licensor features +f,
- $F \subseteq Feat$ is a set of final category features,
- the lexicon *Lex* is a finite subset of $\Sigma^* \times Feat^+$,
- Op := {merge, move} is the set of structure-building operations.

For every MGs it suffices to specify Lex and F.

REG	MGs	Closure Properties	Applications	Conclusion	References
	00000000				
Bare	Phrase St	ructure Trees			

- My definition of *merge* and *move* is tree-based.
- It builds on the notion of Bare Phrase Structure trees and Headedness.

Extended Lexicon

Given a lexicon *Lex*, its *extended lexicon Elex* is the smallest set such that, for $\sigma \in \Sigma^*$, $f \in Feat$, and $\delta \in Feat^*$

• $I \in Lex \rightarrow I \in Elex$

•
$$I := \langle \sigma, f \delta \rangle \in Elex \to I' := \langle \sigma, \delta \rangle \in Elex$$

Bare Phrase Structure Trees (BPS Trees)

The set of *BPS trees over Elex* consists of all strictly binary branching trees over the ranked alphabet $\{<^{(2)}, >^{(2)}\} \cup \{I^{(0)} \mid I \in Elex\}.$

REG 0000	MGs ○●○○○○○○	Closure Properties	Applications	Conclusion O	References
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REG 0000	MGs 00●00000	Closure Properties	Applications	Conclusion O	References
Head	edness				

Headedness

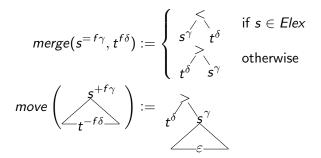
Given a BPS tree t, the *head* of t is given by

$$head(t) := egin{cases} t & ext{if } t \in Elex \ head(t_1) & ext{if } t := \ t_1 & t_2 \ head(t_2) & ext{if } t := \ t_1 & t_2 \ t_1 & t_2 \end{cases}$$

Notation t^{δ} denotes that head(t) carries feature string δ

REG 0000	MGs 000●0000	Closure Properties	Applications	Conclusion O	References
Definin	g Merge	& Move			

Let $\gamma, \delta \in Feat^*$.



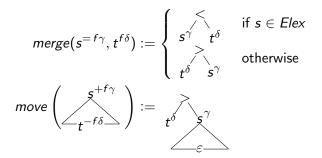
Shortest Move Constraint (SMC)

Every tree $s^{+f\gamma}$ in the domain of *move* has exactly one subtree t such that the first feature of head(t) is -f.

Thanks to the SMC, both Merge and Move are deterministic.

REG 0000	MGs 000●0000	Closure Properties	Applications	Conclusion O	References		
Defining Merge & Move							

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Thanks to the SMC, both Merge and Move are deterministic.

Derived Tree Language

The tree language L(G) derived by MG G with lexicon Lex_G is the largest set of BPS trees such that

- $L(G) \subseteq closure(Lex_G, \{merge, move\}),$
- for every t ∈ L(G), there is some f ∈ F_G such that the feature component of head(t) consists only of f,
- all other leaves have an empty feature component.

Generated String Language

The string language generated by MG G is the string yield of L(G).

Theorem (Harkema 2001; Michaelis 1998, 2001)

MCFGs and MGs are weakly equivalent.

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REG	MGs	Closure Properties	Applications	Conclusion	References
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	v Example	Without Re	ecursion)		

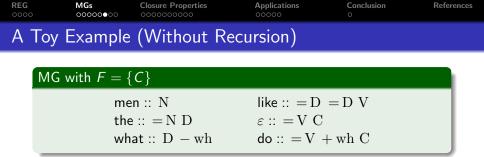
MG with $F = \{C\}$

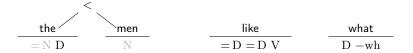
 $\begin{array}{l} \mbox{men} :: \ N \\ \mbox{the} :: \ = N \ D \\ \mbox{what} :: \ D \ - \mbox{wh} \end{array}$

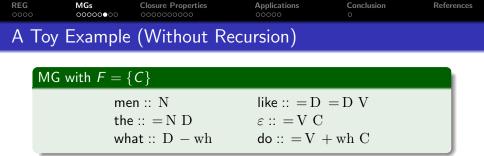
like :: = D = D V ε :: = V C do :: = V + wh C

REG 0000	MGs ○○○○○●○○	Closure Properties	Applications	Conclusion ○	References		
A Toy Example (Without Recursion)							
MG with $F = \{C\}$							
	the	en :: N e :: = N D hat :: D - wh	like :: = D ε :: = V C do :: = V -				

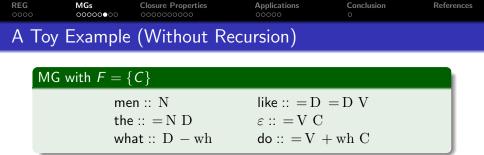


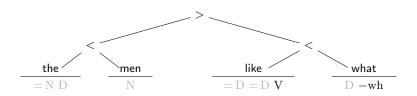


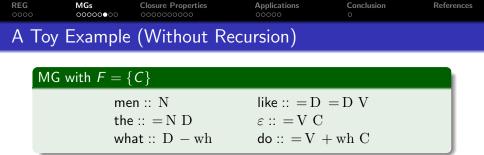


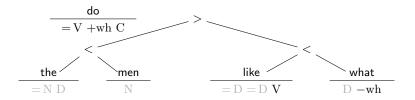


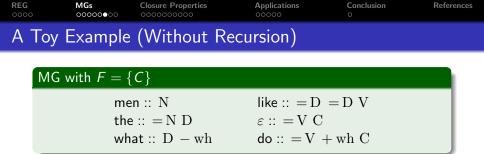


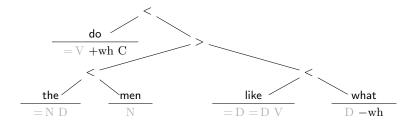


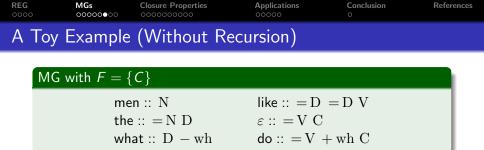


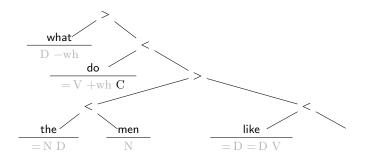








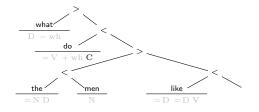


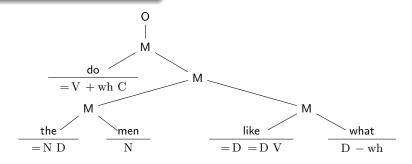


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Deriva	ation Tree	es			

Useful Fact

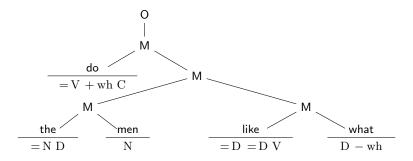
Every MG is fully specified by its set of derivation trees, which is regular (Kobele et al. 2007).





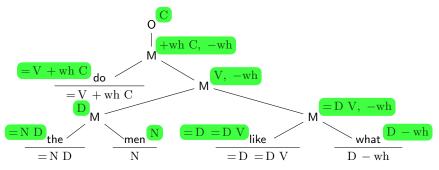


- Defining well-formed derivation trees of MG G only requires keeping track of the feature calculus
 ⇒ deterministic bottom-up automaton with sequences of feature strings as states (and F_A := {⟨f⟩ | f ∈ F_G})
- Due to the SMC, the number of feature strings per state is bounded ⇒ finite number of states





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REG 0000	MGs 00000000	Closure Properties	Applications	Conclusion O	References
Non-Cl	osure Ur	der Intersecti	on with RE	G	

Theorem

The class of MDTLs is not closed under intersection with regular tree languages.

Proof.

- Let ODD contain all trees with an odd number of nodes.
- Let G be the MG given by $F_G = \{c\}$ and Lex_G : a :: a b :: = a = a + k a c :: = a c a :: a - k
- Then there are derivation trees s and t in the closure of Lex_G under {merge, move} that both end in a final category and contain the same lexical items. It is easy to see that s ∈ mder(G') iff t ∈ mder(G') for any MG G', yet s ∉ mder(G) ∩ ODD ∋ t.

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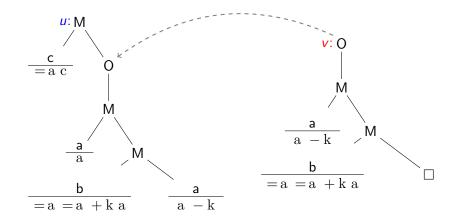
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REG	MGs	Closure Properties	Applications	Conclusion	References
		000000000			
Choic	e of s and	d +			



$$s = u \notin ODD$$

 $t = u + v \in ODD$

REG 0000	MGs 00000000	Closure Properties	Applications	Conclusion O	References
Defini	ng P-Clo	sure			

Projection

Let $\lambda : \Sigma \to \Omega$ be a many-to-one map between alphabets, and π its extension from alphabets to trees. Tree *t* is a *projection* of *s* iff there is a π such that $t = \pi(s)$. The notion extends to tree languages in the natural way.

P[rojection]-Closure

Given a class of languages \mathcal{L} and an operation O, \mathcal{L} is *p*-closed under O iff the result of applying O to some $L \in \mathcal{L}$ is a projection of some $L' \in \mathcal{L}$.

REG	MGs	Closure Properties	Applications	Conclusion	References
		00000000			
D Clo	curo Und	ar Intersection	with REC		

Theorem (REG Intersection P-Closure)

The class of MDTLs over alphabet Σ and features Feat is p-closed under intersection with regular tree languages.

Outline of Proof

- Inspired by Thatcher's theorem (translate recognizable sets into local ones by incorporating states into alphabet)
- Crux: Internal node labels of a derivation tree cannot be refined ⇒ *slices* as a way of relating interior nodes to features on lexical items
- Procedure for refining category and selector features so that they incorporate states of the deterministic bottom-up automaton recognizing regular language

REG 0000	MGs 00000000	Closure Properties	Applications	Conclusion O	References
Slices					

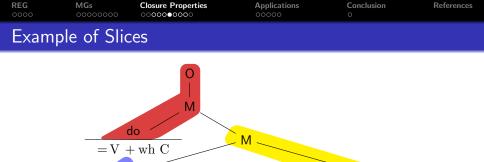
Intuitively, slices are the **derivation tree equivalent of phrasal projection**: Each slice marks the subpart of the derivation that a lexical item has control over by virtue of its selector and licensor features.

Slices

Given a derivation tree t and lexical item l occurring in t, slice(l) is defined as follows:

- $I \in \text{slice}(I)$,
- if node n of t immediately dominates a node s ∈ slice(1), then n ∈ slice(1) iff the operation denoted by the label of n erased a selector or licensor feature of 1.

The unique $n \in \text{slice}(I)$ that isn't (properly) dominated by any $n' \in \text{slice}(I)$ is called the *slice root* of *I*.



Simple Facts About Slices

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= N D

- Every node of a derivation tree belongs to some slice.
- Slices are continuous.

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men

Ν

Moving from slice(*I*) to slice(*I'*) such that *I'* was selected by *I*, one eventually reaches a slice of size 1.

like

= D = D V

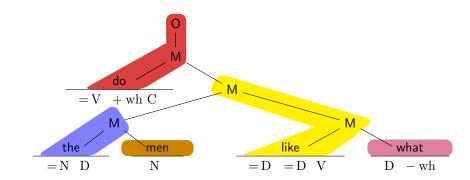
what

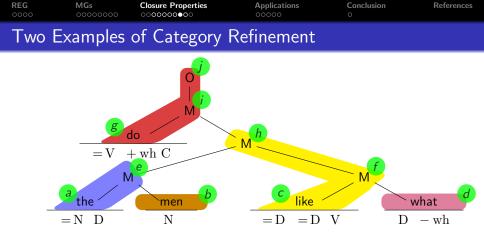
D - wh

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Catego	ory Refin	ement Strate	gy		

- Assume we are given an MG G and deterministic bottom-up automaton A.
- Subscript interior node labels with state of automaton, following Thatcher's strategy.
- Move subscript from slice root of lexical item to its category feature.
- Refine selection features accordingly.
- The set of final categories of the new MG G' is $\{c_q \mid c \in F_G, q \in F_A\}.$
- Note that only finitely many combinations of slices and states need to be considered, so the procedure can be carried out efficiently.



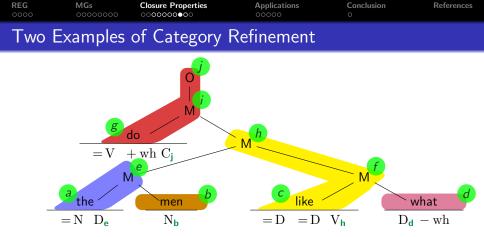




а:: а_о а:: а_о — k

$$\begin{array}{l} \mathsf{b} ::= \mathsf{a}_{\mathsf{o}} = \mathsf{a}_{\mathsf{o}} + \mathsf{k} \; \mathsf{a}_{\mathsf{e}} \\ \mathsf{b} ::= \mathsf{a}_{\mathsf{o}} = \mathsf{a}_{\mathsf{e}} + \mathsf{k} \; \mathsf{a}_{\mathsf{o}} \\ \mathsf{b} ::= \mathsf{a}_{\mathsf{e}} = \mathsf{a}_{\mathsf{o}} + \mathsf{k} \; \mathsf{a}_{\mathsf{o}} \\ \mathsf{b} ::= \mathsf{a}_{\mathsf{e}} = \mathsf{a}_{\mathsf{e}} + \mathsf{k} \; \mathsf{a}_{\mathsf{e}} \end{array}$$

$$c ::= a_o c_o$$

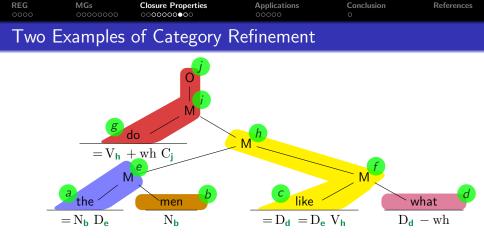


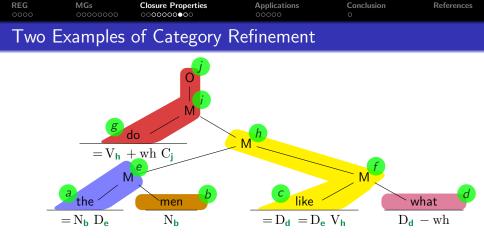
a:: a_o

a :: a_o - k

$$b ::= a_o = a_o + k a_e b ::= a_o = a_e + k a_o b ::= a_e = a_o + k a_o b ::= a_e = a_e + k a_e$$

 $c :: = a_0 c_0$





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Correc	tness of	Procedure			

- Suppose that $mder(G') \neq mder(G) \cap L(A)$.
- Then there must be some tree t such that $t \in mder(G')$ iff $\pi(t) \notin mder(G) \cap L(A)$. So head(t) has a category feature c_q , but A does not assign state q to the root of $\pi(t)$.
- Since A is deterministic, such a situation may arise only if A entered the slice in a state that differs from the subscripts on the corresponding selector feature of head(t).
- By induction on slices, we eventually reach a slice of size 1 to which A assigned a state that differs from the subscript of the category feature of its lexical item. But A is deterministic. Contradiction.

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P-Closure Corollaries

- The class of **MDTLs over** Σ , *Feat* is p-closed under
 - intersection,
 - relative complement.
- Given lexicon *Lex*, the class of **MDTLs over subsets of** *Lex* is p-closed under
 - complement,
 - union.
- For every regular tree language L and linear transduction τ with an MDTL as its co-domain, it holds that τ(L) is a projection of some MDTL.

REG MGs Closure Properties Applications Conclusion References Minimalist Grammars with Regular Control

Minimalist Grammars with Regular Control (MGRCs)

An MG is a 6-tuple $G := \langle \Sigma, Feat, F, Lex, Op, \mathcal{R} \rangle$, where

- Σ , Feat , F, Lex, and Op are defined as usual, and
- \mathcal{R} is a finite collection of regular tree languages.

Its controlled derivation tree language is $cder(G) := mder(G) \cap \mathcal{R}$. The derived tree language of G (and its string yield) are obtained from cder(G) via the mbutt of Kobele et al. (2007).

- MGRCs are more succinct than their refined equivalent.
- Given a lexicon Lex and n ≥ 0, let Lex⁽ⁿ⁾ := {I ∈ Lex | I has exactly n selector features}. In the worst case

$$|Lex_{G'}| = \sum_{i \ge 0} \left(|Lex_G^{(i)}| \cdot |Q|^{i+1} \right)$$

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Minimalist Grammars with Regular Control (MGRCs)

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		Reference-Se			
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- Reference-set constraints are **economy conditions** similar to OT: Given some input tree *t*
 - compute the set of competing output candidates,
 - rank them according to some economy metric,
 - discard all sub-optimal candidates.
- Graf (2010a,b): Most reference-set constraints in the syntactic literature can be modelled by linear tree transductions. In particular, those **constraints act as filters**, so the transductions have MDTLs as their domain and co-domain.
- From the previous corollary for linear transductions it follows that **the expressivity of MGs is not increased**.



- Expletive constructions in English show subject-verb agreement even though no movement seems to be involved.
 - (1) a. There seems to be a man in the garden.
 - b. There seem to be **three men** in the garden.
- The subject position is filled by the expletive
 - \Rightarrow arguably no movement

Proposal

Regular constraint operating on "pseudo-features" (not part of the MG itself) \Rightarrow enforce non-local dependencies without movement

• Quite generally, this allows us to **enrich MGs with AGREE** (Chomsky 2000).

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Applic	ation 3:	Relativized M	linimality		

- In Minimalist syntax, the contrast below is explained by *Relativized Minimality*: If a movement licensor feature can be checked by two different phrases, **the closer one moves**.
 - (2) Who/what bought t who/what?
 - (3) * Who/What bought who/what *t*?
- Relativized Minimality relies on both who and what carrying a -wh feature. This idea conflicts with the SMC, and in order to derive (2), we must allow who/what to appear without a -wh-feature. But then nothing in the MG blocks (3).

Proposal

Moving phrase XP with feature -f to ZP is banned if there is a closer YP with pseudo feature -f.

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Further Applications								

- Island constraints
- Phases
- that-trace filter
- L-marking
- Limited feature percolation/Pied-Piping
- Control/Binding(?)

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Conclusion							

- MDTLs are not closed under intersection with regular tree languages.
- However, they enjoy **p-closure properties akin to regular languages**:
 - intersection,
 - intersection with regular tree languages,
 - union,
 - (relative) complement,
 - certain linear transductions.
- Hence, enriching MGs with regular control over their derivations does not increase their generative capacity.
- Numerous applications; in particular, ideas from model-theoretic syntax can be easily incorporated

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