

# Formal Processing Theory

or

## Parsing Without Parsers

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# The Take Home Messages

- Formal parsing models of processing are worth pursuing.
- **But:** problem of too many solutions
- Our approach is too fine-grained.
- We need a more general perspective.
- We need
  - abstraction
  - theorems
  - proofs

# Outline

- 1 Why Care About Syntactic Processing?
- 2 Top-Down Parsing of Minimalist Grammars
- 3 Memory-Based Processing Predictions
- 4 Towards a Proof-Based Approach to Processing
  - Embedding Invariance
  - Isolated Embeddings
  - Informal Observations on Other Rankings
  - Movement, oh Movement!

# Parsing $\neq$ Processing

- A grammar without an efficient parser is useless  
⇒ parsing is an important research area
- But syntactic processing is only about **the human parser**, with all its warts and quirks:
  - small working memory,
  - no full parallelism or memoization,
  - garden paths,
  - grammaticality illusions,
  - merely local syntactic coherence effects,
  - :
- From an engineering perspective, the human parser is terribly flawed (neither sound nor complete).
- So why should we care about modelling the human parser when CYK, Earley & Co are much more sophisticated?

# Why Syntactic Processing Matters

## ① Applications

- *Performance*  
Despite memory limitations, the human parser outperforms our fastest parsers (better than linear time).
- *Future applications*  
Once you have a very expressive text generation system, you must ensure that its output is processable.

## ② Theory

- *Inherent interest*  
Every aspect of language is ripe for mathematical inquiry.
- *Building bridges to other fields*  
We've got a great toolkit, let's show the world what it can do!
- *Clues about strong generative capacity*  
Processing effects provide **clues about syntactic structure**.

# A Recent Attempt to Link Processing and Syntax

- **Stabler (2011, 2013)**

- top-down parser for full class of Minimalist grammars
- can handle virtually all analysis in the generative literature

- **Kobele et al. (2012)**

- memory-usage metric relates parser behavior to processing
- processing predictions are highly dependent on syntactic analysis (e.g. head VS phrasal movement)

# The Most Informal Intro to MGs Ever

Minimalist grammars treat syntax like chemistry.

<b>Chemistry</b>	<b>Syntax</b>
atoms	words
electrons	features
molecules	sentences

- Every word is a collection of features.
- Every feature has either positive or negative polarity.
- Features of opposite polarity annihilate each other.
- Feature annihilation drives the structure-building operations **Merge** and **Move**.

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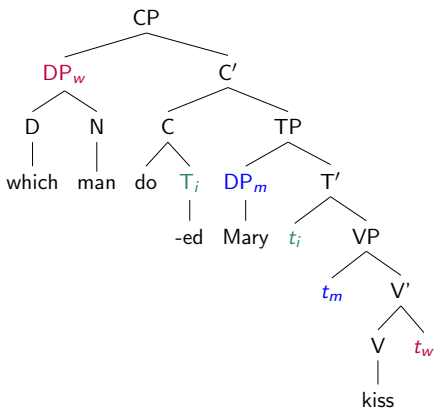
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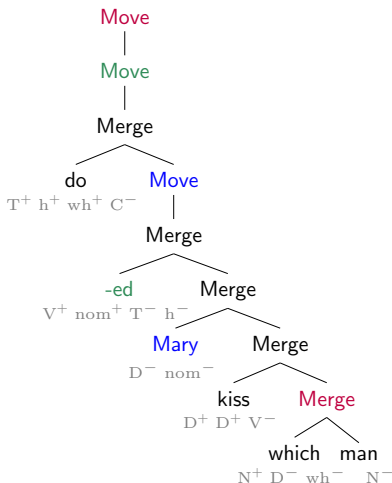
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# MGs in Action



Phrase Structure Tree



Derivation Tree

## Some Important Properties

- MGs are weakly equivalent to MCFGs and thus mildly context-sensitive. (Harkema 2001; Michaelis 2001)
- But we can decompose them into two finite-state components: (Michaelis et al. 2001; Kobele et al. 2007; Mönnich 2006)
  - a regular language of well-formed derivation trees
  - an MSO-definable mapping from derivations to phrase structure trees
- **Remember:** Every regular tree language can be reencoded as a CFG (with more fine-grained non-terminal labels). (Thatcher 1967)

### The Context-Free Backbone of MGs

MGs can be viewed as CFGs with a more complicated mapping from trees to strings.

# The Top-Down MG Parser

- **Core Idea**

**recursive descent parser** over context-free derivation trees

- top-down
- depth-first
- left-to-right

- **Essential Modification**

linear order in the derivation tree does not correspond to linear order in the string

⇒ “left-to-right” refers to string order, not tree order

- **Bells and Whistles**

- parser hooks directly into lexicon and feature calculus
- beam search weeds out unlikely parses
- constraints on movement reduce parsing complexity

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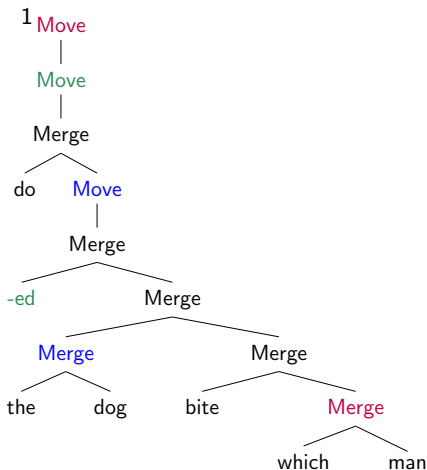
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at which step the node is conjectured

- **Outdex**

at which step the parser considers the node done



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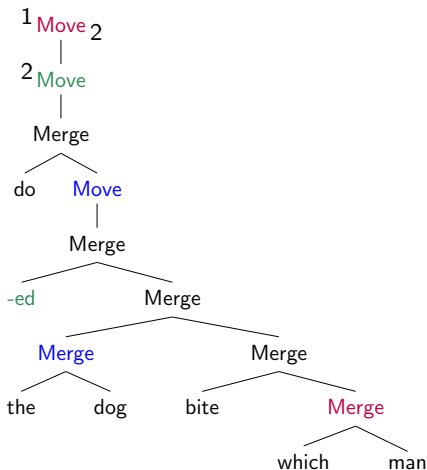
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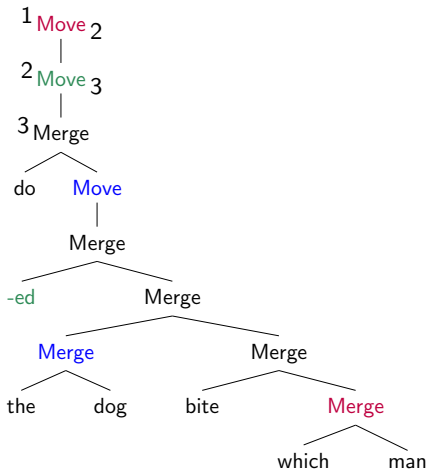


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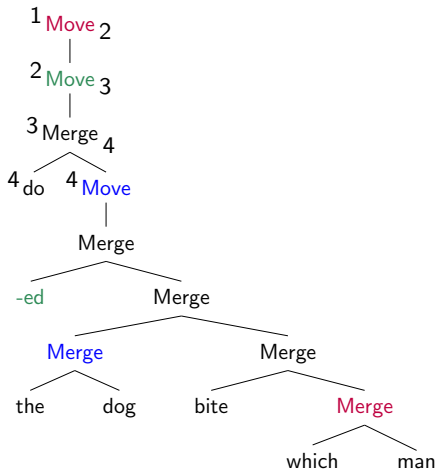
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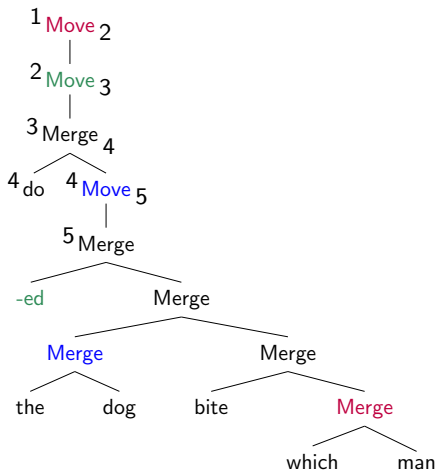
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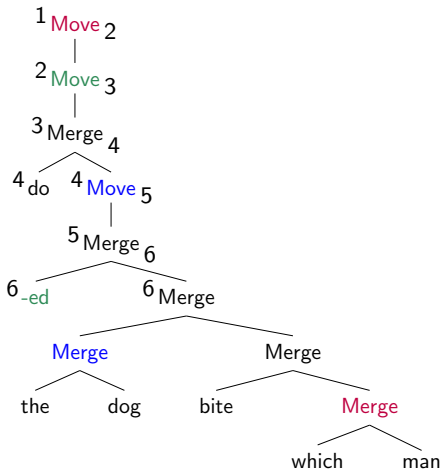


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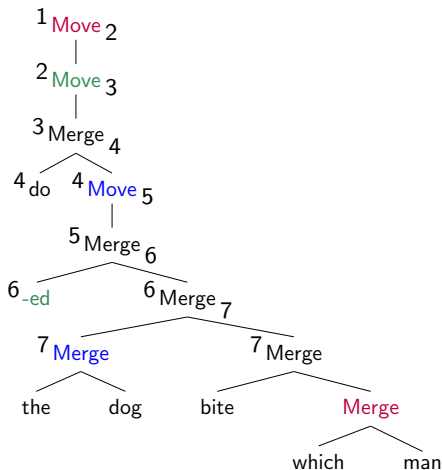
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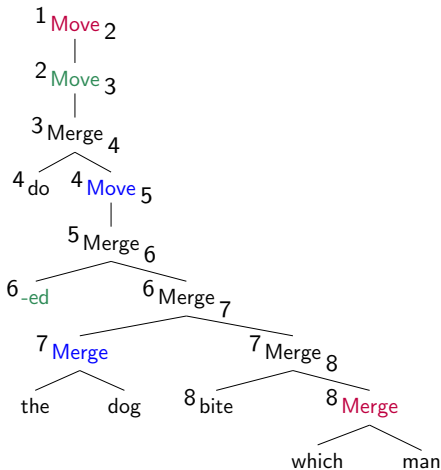


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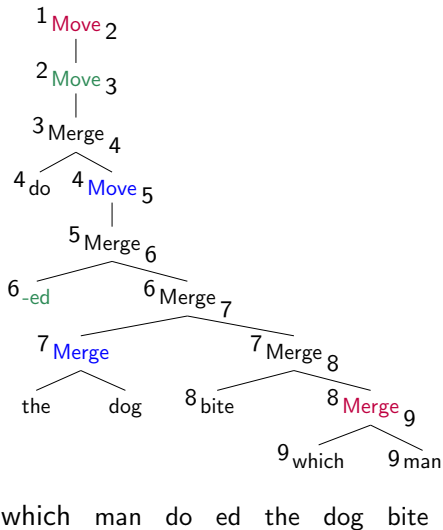
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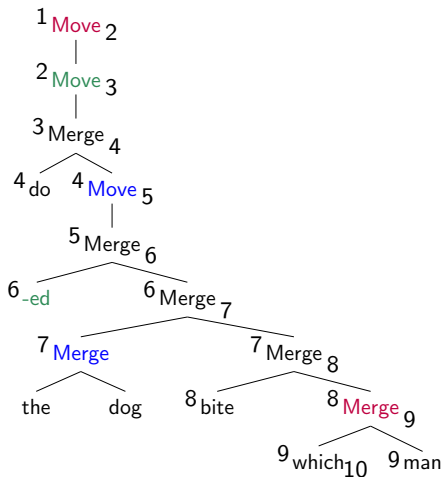
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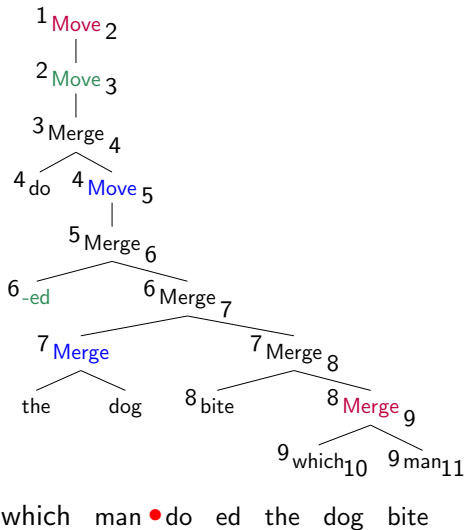
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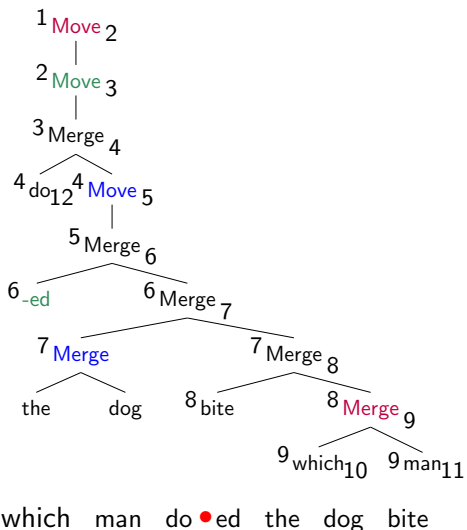


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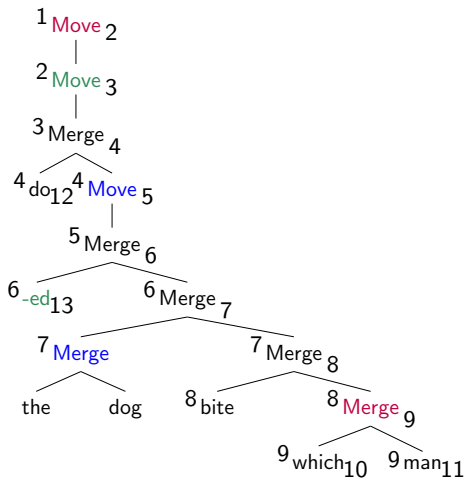
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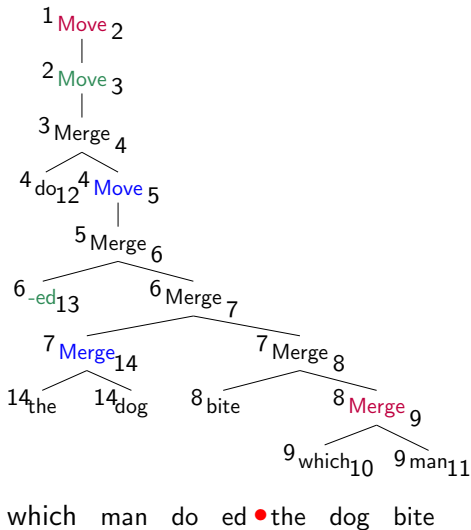




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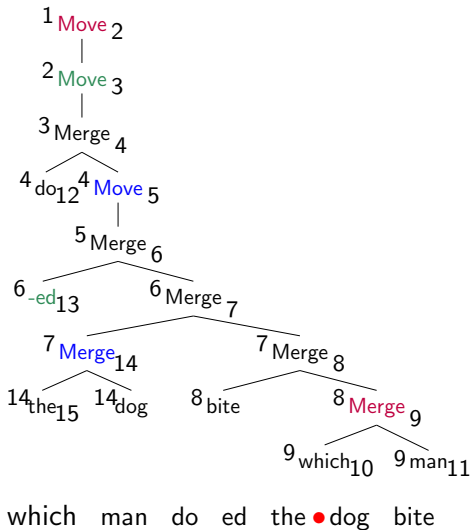
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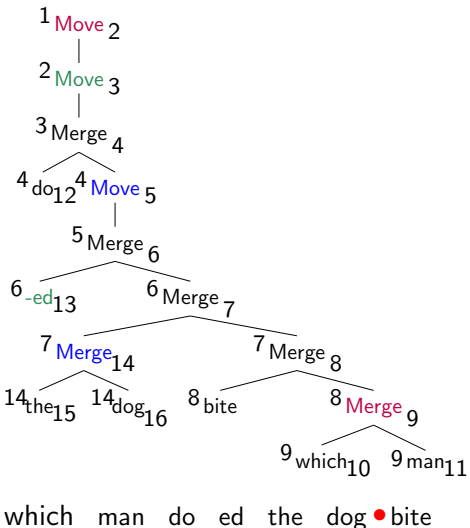
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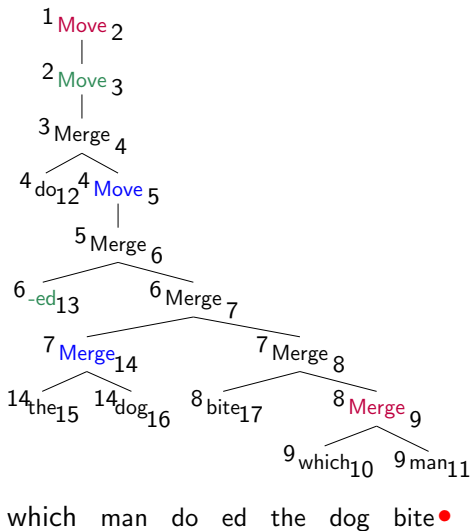
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# Relating Parsing and Processing

- **General Approach** (Kobele et al. 2012; Graf and Marcinek 2014; Graf et al. 2015)
  - pick competing syntactic analyses
  - pick metric to relate parsing behavior to processing difficulty
  - see which analysis gets it right
- **Simplifying Assumption**
  - consider only parser's behavior for correct parse
  - factors out problem of finding correct parse
- **Appeal**
  - maximally simple
  - MGs allow for explicit, linguistically sophisticated analyses
  - fully specified parsing model with precise predictions

# Notions of Memory Usage

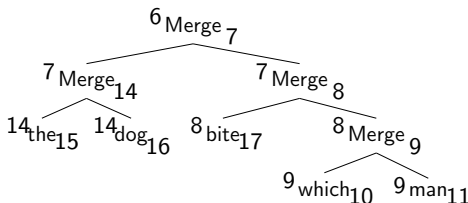
All metrics studied so far build on **memory usage**.

(cf. Gibson 1998)

**Tenure** how long a parse item ( $\approx$  node)  $p$  is stored  
 $outdex(p) - index(p)$

**Payload** how many parse items were stored during the parse  
 $|\{p \mid outdex(p) - index(p) > 2\}|$

**Gap** size of parse items  $\approx$  distance of movement



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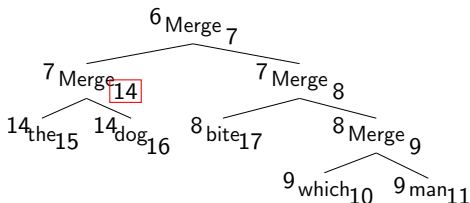
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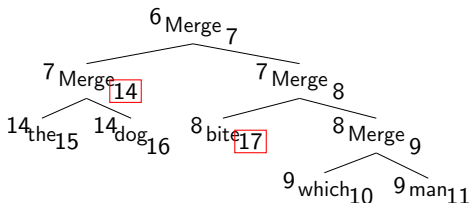
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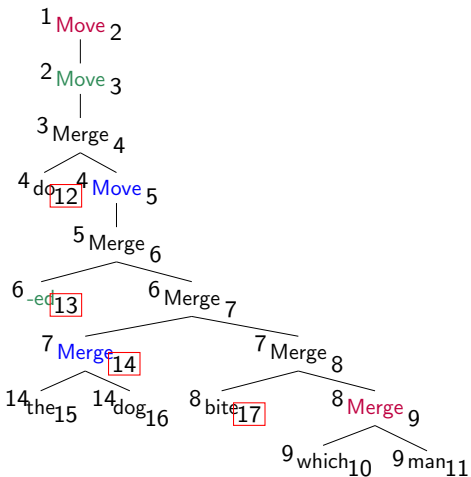


# Memory-Based Metrics of Processing Difficulty

- Max** highest tenure in parse  
 $\max(\{t \mid t \text{ is the tenure of some node } n\})$
- Max<sup>R</sup>** vector of tenure for all nodes, in decreasing order
- Box** payload of parse  
 $|\{n \mid n \text{ is a node with tenure } > 2\}|$
- Sum** summed tenure of payload  
 $\sum_{n \text{ has tenure } > 2} \text{tenure-of}(n)$

# Example Values for Each Metric

<b>Max</b>	9
<b>Max<sup>R</sup></b>	$\langle 9, 8, 7, 7, 2, \dots \rangle$
<b>Box</b>	4
<b>Sum</b>	31



# Processing Phenomena: Embedding

- Left embedding is easy
  - (1) John's father's cousin's house's roof collapsed.
- Center embedding is hard, right embedding is easy
  - (2)
    - a. The cheese that the mouse that the cat chased ate was rotten.
    - b. The cheese was rotten that the mouse ate that the cat chased.
- Crossing dependencies are easier than nested dependencies.
  - (3)
    - a. that John Mary Peter swim teach let. (German)
    - b. that John Mary Peter let teach swim. (Dutch)

# Sentential Clauses and Relative Clauses

- A relative clause inside a sentential clause is easy.
  - (4) The fact **that the employee who the manager hired stole office supplies** worried the executive.
- A sentential clause inside a relative clause is hard.
  - (5) The executive **who the fact that the employee stole office supplies worried** hired the manager.

# Subject and Object Relative Clauses in English

Subject relative clauses (SRCs) are easier than object relative clauses (ORCs).

- (6) a. **The reporter** who \_\_ attacked the senator admitted the error.
- b. **The reporter** who the senator attacked \_\_ admitted the error.

# RCs in East Asian

RCs **precede the modified noun** in Chinese, Japanese, Korean.  
SRC is still preferred over ORC.

## (7) *Chinese*

- a. \_\_\_ attacked the senator who **reporter** admitted the error.
- b. the senator attacked \_\_\_ who **reporter** admitted the error.

In addition, Korean and Japanese also have SOV order.

## (8) *Korean*

- a. \_\_\_ the senator attacked who **reporter** admitted the error.
- b. the senator \_\_ attacked who **reporter** admitted the error.

# Overview of Findings

## Methodology

- ① take derivations for sentences with processing contrast
- ② compute indices and outdices
- ③ compute value according to chosen metric
- ④ easier sentence should have lower value

	Max	Max <sup>R</sup>	Sum	Box
Center/Right	✓	✓	✓	✓
Center/Crossing	✓	✓	≈	≈
Left embedding	×	×	×	≈
SC/RC vs RC/SC	≈	✓	✓	✓
SRC vs ORC (Eng)	≈	✓	✓	✓
SRC vs ORC (Asian)	≈	×	×	×

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# Predictions for East Asian RC-Processing

		Promotion			Wh-Movement		
		all	lex.	pron.	all	lex.	pron.
<i>Korean</i>	<b>Max</b>	tie	tie	tie	tie	tie	tie
	<b>Max<sup>R</sup></b>	ORC	ORC	ORC	ORC	ORC	ORC
	<b>Sum</b>	ORC	ORC	ORC	ORC	ORC	ORC
	<b>Box</b>	tie	ORC	ORC	ORC	ORC	ORC
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	<b>Max<sup>R</sup></b>	ORC	ORC	ORC	ORC	ORC	ORC
	<b>Sum</b>	SRC	ORC	ORC	tie	ORC	ORC
	<b>Box</b>	SRC	SRC	tie	SRC	tie	ORC

# Why Modelling is not Enough

Parameters of the modelling approach...

- 1 Syntactic analysis
- 2 Parser/Node Indexation algorithm
- 3 Processing difficulty metric

... and a swath of problems

- infinitely many choices for each parameter
- complex and unpredictable interaction
- solution underspecified by evidence

## Solution

What we need are the standard tools of mathematical linguistics:

- precisely defined yet general properties,
- proofs instead of simulations,
- theorems about infinite classes of parsers/metrics

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# Metric Property 1: Embedding Invariance

A metric  $M$  is **embedding invariant** iff



## Psycholinguistic Motivation

Many contrasts are independent of the containing clause:

- SC/RC vs RC/SC
- SRC vs ORC
- Center embedding vs right embedding
- Nested vs crossing dependencies

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# Shape-Blind

## Definition

Two subtrees are

- **feature-equivalent** iff their list of unchecked features is identical.
- **$M$ -equivalent** with respect to metric  $M$  iff  $M$  assigns them the same value.

## Definition (Shape-Blind)

A metric  $M$  is **shape-blind** iff it holds that



if **a** and **c** are feature-equivalent and  $M$ -equivalent.

# Embedding Invariance Implies Shape-Blindness

## Theorem

*A metric  $M$  is embedding invariant only if it is shape-blind.*

## Lemma

*Max and Gap are not shape-blind.*

## Proof.

- **Max**: size of left subtree determines tenure of its right sibling
- **Gap**: movement paths can differ in length □

## Corollary

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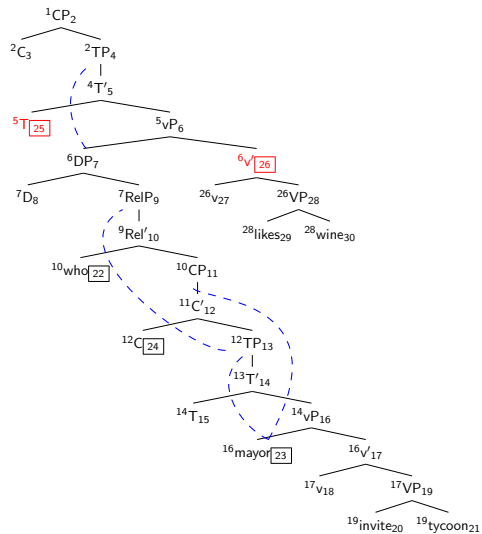
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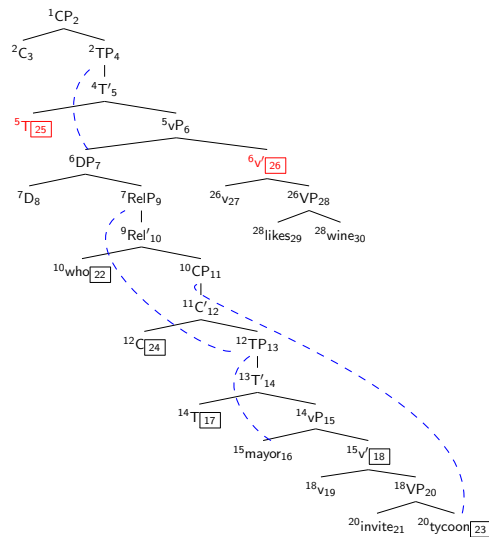
# Explaining the Failure of Max for Chinese SRC/ORC



## Intuition

Embedding the DPs in their clauses causes high tenure. This outweighs all SRC/ORC differences.

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# Isolated Embeddings

## Definition (Isolation)

A subtree is isolated iff the only unchecked feature is the category feature of its root.

## Theorem

*Every “reasonable” shape-blind metric is embedding invariant for isolated subtrees.*

# Other Rankings are Embedding Invariant

## Theorem

**Box**, **Gap**, and **Sum** are invariant under isolated embeddings.

## Proof.

- An isolated embedding of **a** into **b** only adds a constant number  $n$  of tenure nodes, where  $n$  depends only on **b**.
- This guarantees that the value of a derivation under the respective metric is only increased by a constant amount that is a function of  $n$  and the choice of metric. □
- The East Asian RC cases can be analyzed as isolated embeddings of distinct DPs into the same matrix clause.
- So why do most of these metrics fail nonetheless?

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# The Role of Movement

## Definition (Move Power)

The **Move power** of a derivation is the number of precedence relations that are altered by Move.

## Definition (Surface orientation)

A metric  $M$  is **surface-oriented** iff it holds for all trees  $a$  and  $c$  that

- if  $a$  and  $c$  are identical *modulo* Move, and
- the Move power of  $a$  is less than the Move power of  $c$ , then
- $M(a) \leq M(c)$ .

## Theorem

**Max**, **Box**, and **Sum** are surface oriented. **Gap** is not.

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## Definition (Surface orientation)

A metric  $M$  is **surface-oriented** iff it holds for all trees  $a$  and  $c$  that

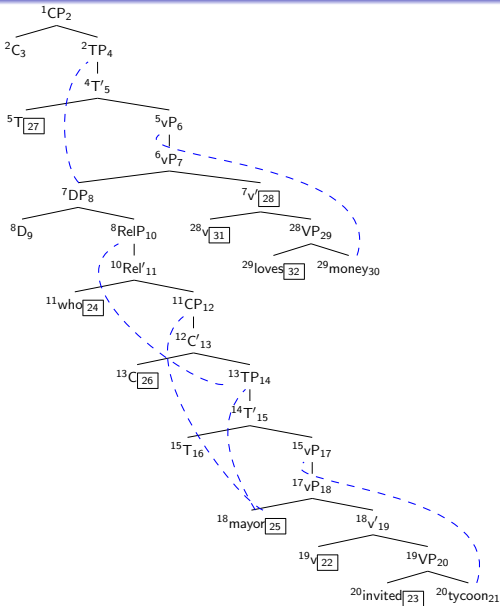
- if  $a$  and  $c$  are identical *modulo* Move, and
- the Move power of  $a$  is less than the Move power of  $c$ , then
- $M(a) \leq M(c)$ .

## Theorem

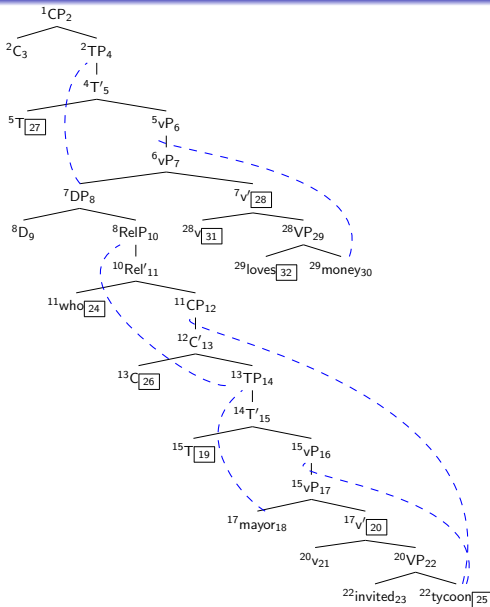
**Max**, **Box**, and **Sum** are surface oriented. **Gap** is not.



# ORC Preference in Korean



# ORC Preference in Korean



# Which Properties do we Want?

- **Embedding Invariance**  
mostly yes, but some apparent exceptions
- **Isolated Embedding Invariance**  
yes
- **Surface-Oriented**  
mostly no?

# The Bigger Picture

- Modeling provides important clues, but it is not enough.
- Modeling cannot provide a formal theory of what properties an adequate processing metric need to satisfy.
- We need to think in terms of more abstract and general properties like embedding invariance.
- We may never find a unique solution to the processing problem due to insufficient evidence, but we can try to characterize the (infinite?) class of viable solutions.

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